

**Title: Testing for Measurement Invariance with Latent Class Analysis****Authors:** Miloš Kankaraš\*, Guy Moors\*, and Jeroen K. Vermunt\***Abstract**

Testing for measurement invariance can be done within the context of multigroup latent class analysis. Latent class analysis can model any type of discrete level data, which makes it an obvious choice when nominal indicators are used or when a researcher's aim is at classifying respondents in latent classes. The multigroup latent class (LC) model can be specified in three different ways, i.e. by adopting a probabilistic, a log-linear or a logistic parameterization. We define and compare these different forms of parameterization. The starting point is the standard LC model in which indicators and latent variables are defined at the nominal level. Additionally, we focus on LC models with ordinal indicators as well as LC factor models with ordinal indicators. Testing for measurement invariance involves estimating LC models with different degrees of homogeneity. We explain the procedure for investigating measurement invariance at both the scale as well as the item level. We illustrate the approach with two examples. The first example is a multigroup LC analysis with nominal indicators; the second a multigroup LC factor analysis with ordinal indicators.

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## INTRODUCTION

There are three important reasons why latent class analysis offers a valuable approach for testing measurement invariance in cross-cultural survey research. First, latent class (LC) analysis can be used to identify latent structures from the relationships among sets of discrete observed variables, and the questions used in survey research have almost always discrete (ordinal or nominal) response categories. Second, different from – for instance – the more popular multigroup confirmatory factor analysis (MCFA), which has been elaborated in the previous chapters, LC models can treat latent variables as nominal – e.g. to identify a typological classification from a given set of categorical indicators – as well as ordinal – e.g. to investigate the scalability of a set of categorical indicators. These two specifications are sometimes referred as LC cluster and LC factor models, respectively. Third, multigroup LC analysis offers a flexible alternative to the more commonly used MCFA and multigroup item response theory (IRT) approaches, which both rely on stronger distributional assumptions than LC analysis.

This chapter is organized as follows. We first introduce the basic multigroup LC model, where attention is paid to three possible parameterizations of the model, and subsequently discuss two important extensions of the basic model; i.e. extension for dealing with ordinal indicators and for modeling the latent variables as ordinal variables. We then turn to the analysis of measurement invariance using multigroup LC models, discussing the general procedure, as well as methods for parameter estimation and evaluation of model fit. Finally, two examples are presented in which multigroup LC cluster and LC factor models are applied to a set of nominal and ordinal observed variables, respectively, with the aim to assess measurement invariance in a cross-cultural comparative setting.

## 1. MULTIGROUP LATENT CLASS MODELS

The multigroup extension of the standard LC model has been developed for the analysis of latent structures of observed categorical variables across two or more groups (Clogg & Goodman, 1984; 1985). When comparing latent structures across groups, a number of possible outcomes can occur: they may turn out to be completely different (heterogeneous model), partially different (partially homogeneous model), or completely the same (homogeneous model). In this section we focus on the heterogeneous model in which all of model parameters are group-specific. We will discuss the classical probabilistic parameterization of the multigroup LC model, as well as its log-linear and logistic parameterization. We also pay attention to multigroup LC models for ordinal responses and with ordinal latent variables. The next section, which deals with the application of the multigroup LC model in cross-cultural research, discusses models in which some or all parameters are restricted to be equal (invariant) across groups.

### 1.1 The heterogeneous LC model

Multigroup LC models assume the presence of three types of categorical variables: observed (indicator) variables; an unobserved (latent) variable that accounts for the relationships between the observed variables; and a grouping variable  $G$ , which is a categorically-scored, manifest variable that can be associated with both the indicators and the latent variable.

Let us assume a LC model with four observed polytomous variables  $A$ ,  $B$ ,  $C$ , and  $D$  having  $I$  ( $i=1, 2, \dots, I$ ),  $J$  ( $j=1, 2, \dots, J$ ),  $K$  ( $k=1, 2, \dots, K$ ), and  $L$  ( $l=1, 2, \dots, L$ ) categories respectively; one latent polytomous variable  $X$  with  $T$  classes ( $t=1, 2, \dots,$

T), and one grouping variable G with S groups indexed by  $s=1, \dots, S$ . The variables A, B, C and D are observed in each of these S groups. Thus, we have a set of S four-way ( $I \times J \times K \times L$ ) observable contingency tables, or one five-way table ( $I \times J \times K \times L \times S$ ). Then the multigroup LC model takes the following form:

$$\pi_{ijklts}^{ABCDX|G} = \pi_{ts}^{X|G} \pi_{its}^{A|XG} \pi_{jts}^{B|XG} \pi_{kts}^{C|XG} \pi_{lts}^{D|XG} \quad (1)$$

Here,  $\pi_{ijklts}^{ABCDX|G}$  denotes the conditional probability that an individual who belongs to the  $s$ th group will be at level  $(i, j, k, l, t)$  with respect to variables A, B, C, D, and X. The conditional probability of X taking on level  $t$  for a member of the  $s$ th group is denoted with  $\pi_{ts}^{X|G}$ , which determines the LC proportion for the  $s$ th group.  $\pi_{its}^{A|XG}$  is the conditional probability of an individual taking level  $i$  of variable A, for a given level  $t$  of the latent variable X and for a given group membership  $s$  of the grouping variable G. Parameters  $\pi_{jts}^{B|XG}$ ,  $\pi_{kts}^{C|XG}$ , and  $\pi_{lts}^{D|XG}$  are similarly defined conditional probabilities.

It should be noted that Equation 1 implies that indicator variables A, B, C, and D are independent from each other, given the value of the latent variable X. This is usually referred to as the assumption of local independence (Lazarsfeld & Henry, 1968). The latent class and conditional response probabilities are constrained to sum to 1:  $\sum_t \pi_{ts}^{X|G} = 1$  and  $\sum_i \pi_{its}^{A|XG} = 1$ , etc.

The model presented in Equation 1 can be called a heterogeneous model since all model parameters differ across groups. In fact, it is equivalent to applying a standard LC model (see Equation 2) for each group separately (Clogg & Goodman, 1985). Or alterably, the standard LC model can be viewed a special case of the more general multigroup LC model (Equation 1) with the number of groups  $S=1$ .

$$\pi_{ijklt}^{ABCDX} = \pi_t^X \pi_{it}^{A|X} \pi_{jt}^{B|X} \pi_{kt}^{C|X} \pi_{lt}^{D|X} \quad (2)$$

The probabilistic LC model presented in Equation 1 can also be parameterized using log-linear terms (Haberman, 1979; Goodman, 1974; McCutcheon, 2002). The conditional response probabilities from the probabilistic parameterization can be obtained from log-linear terms as follows (Haberman, 1979; Heinen, 1996):

$$\pi_{its}^{A|XG} = \frac{\exp(\lambda_i^A + \lambda_{it}^{AX} + \lambda_{is}^{AG} + \lambda_{its}^{AXG})}{\sum_i \exp(\lambda_i^A + \lambda_{it}^{AX} + \lambda_{is}^{AG} + \lambda_{its}^{AXG})}, \text{ etc.} \quad (3)$$

While  $\lambda_i^A$  and  $\lambda_{it}^{AX}$  represent the parameters of the standard, single-group LC model,  $\lambda_{is}^{AG}$  and  $\lambda_{its}^{AXG}$  are the log-linear parameters that depict the inter-group variability of the two former parameters. Parameters  $\lambda_{its}^{AXG}$  are sometimes referred to as ‘interaction effects’ as it indicates that the latent and grouping variable interact with each other in their effect on the indicator variable. In other words, the relationship between item responses and latent variables is modified by the group membership. In a similar manner,  $\lambda_{is}^{AG}$  refers to a ‘direct effect’ of the grouping variable G on the indicator A. Such direct effects are present when group differences in item responses can not fully be explained by group differences in the latent factors – that is when the group variable influences indicators independently of the latent variable.

Not only the response probabilities, but also the class membership probabilities  $\pi_{ts}^{X|G}$  can be defined in terms of log-linear parameters; that is,

$$\pi_{ts}^{X|G} = \frac{\exp(\gamma_t^X + \gamma_{ts}^{XG})}{\sum_{t=1}^T \exp(\gamma_t^X + \gamma_{ts}^{XG})} \quad (4)$$

where the symbol  $\gamma$  denotes a log-linear parameter of the marginal distribution of the latent variable X (Magidson and Vermunt, 2001).

Above we presented two possible parameterizations of the multigroup LC model, which we called the probabilistic and the log-linear parameterization. A third

way of specifying a multigroup LC models is by using a logistic regression-type of equation for the item response probabilities. In this logistic parameterization, the model for indicator item A takes on the following form:

$$\pi_{its}^{AXG} = \frac{\exp(\alpha_{is}^{AG} + \beta_{its}^{AXIG})}{\sum_i \exp(\alpha_{is}^{AG} + \beta_{its}^{AXIG})}, \text{ etc.} \quad (5)$$

where  $\alpha_{is}^{AG}$  represent the group-specific intercepts and  $\beta_{its}^{AXIG}$  the group-specific slope parameters. The slope parameter  $\beta_{its}^{AXIG}$  indicates the strength of the relationships between the latent variable and the indicator variable and can thus be interpreted as a factor loading expressed in log-linear terms (Vermunt and Magidson, 2005). Note that there is a straightforward connection between the log-linear and the logistic formulations of the multigroup LC model presented in Equations 3 and 5:

$$\alpha_{is}^{AG} = \lambda_i^A + \lambda_{is}^{AG} \quad (6)$$

and

$$\beta_{its}^{AXIG} = \lambda_{it}^{AX} + \lambda_{its}^{AXG} \quad (7)$$

In their unrestricted form, the three parameterizations of the multigroup LC model are essentially equivalent, estimating the same number of parameters and producing identical expected values. However, they allow for slightly different types of model restrictions which have important implications for the procedures to test measurement equivalence. First, in the probabilistic parameterization, equivalence is studied by restricting probabilities to be group invariant, in the loglinear parameterization by eliminating interaction and direct effects, and in logistic formulation by restricting intercepts and slopes to be invariant. Second, the latter two parameterizations are needed to formulate models in which indicator or latent

variables are treated as discrete-ordinal. The next two sections focus on ordinal indicators and ordinal latent variables.

## 1.2 Multigroup LC models with ordinal indicators

As noted previously, the log-linear and logistic parameterizations of the LC model allow for the formulation of restricted LC models for ordinal observed and ordinal latent variables. This is achieved by introducing linear restrictions among the parameters for the different categories of the same variable. When applied to the observed variables, these linear restrictions define them as discrete-ordinal variables. This is an important extension of the LC model since in many areas of social sciences, including cross-cultural comparative research, indicator items are often of a discrete-ordinal form (e.g., rating scales).

One straightforward way to define ordinal indicator variables is to assume equidistance between their categories and to modify the log-linear and logistic models for nominal items defined in Equations 3 and 5 by using equidistant category scores. The resulting response model takes on the form of an adjacent-category ordinal logit model. For example, in the case indicator item A is a 5-point rating scale ( $i=1, 2, \dots, 5$ ) these scores could be:

$$v_i^A = i = \{1 \text{ if } i=1, 2 \text{ if } i=2, \dots, 5 \text{ if } i=5\} \quad (8)$$

In the log-linear specification of Equation 3, the  $v_i^A$  are used to restrict  $\lambda_{it}^{AX}$ , as well as the direct and interaction effects as (see, e.g., Heinen, 1996)

$$\lambda_{it}^{AX} = v_i^A \lambda_{*t}^{AX}, \lambda_{is}^{AG} = v_i^A \lambda_{*s}^{AG}, \text{ and } \lambda_{its}^{AXG} = v_i^A \lambda_{*ts}^{AXG}, \quad (9)$$

and the intercepts and slopes of the logistic model defined in Equation 5 as

$$\beta_{its}^{AXIG} = v_i^A \beta_{*ts}^{AXIG}. \quad (10)$$

Depending on the parameterization, the conditional response probability for ordinal indicator A becomes

$$\pi_{it}^{AIX} = \frac{\exp(\lambda_i^A + v_i^A \lambda_{*t}^{AX} + v_i^A \lambda_{*s}^{AG} + v_i^A \lambda_{*ts}^{AXG})}{\sum_i \exp(\lambda_i^A + v_i^A \lambda_{*t}^{AX} + v_i^A \lambda_{*s}^{AG} + v_i^A \lambda_{*ts}^{AXG})}, \quad (11)$$

or

$$\pi_{its}^{AIXG} = \frac{\exp(\alpha_{is}^{AIG} + v_i^A \beta_{*ts}^{AXIG})}{\sum_i \exp(\alpha_{is}^{AIG} + v_i^A \beta_{*ts}^{AXIG})}. \quad (12)$$

In Equation 11, the loading for variable A on the latent variable X is given by  $\lambda_{*t}^{AX}$ , and  $\lambda_{*ts}^{AXG}$  indicates how it differs across groups. In Equation 12,  $\beta_{*ts}^{AXIG}$  is the group-specific loading parameter, where  $\beta_{*ts}^{AXIG} = \lambda_{*t}^{AX} + \lambda_{*ts}^{AXG}$ . It can easily be observed that for ordinal indicators the two parameterizations are no longer equivalent with respect to the part concerning the intercepts and the direct effects. This can be seen by writing  $\alpha_{is}^{AIG} = \lambda_i^A + v_i^A \lambda_{*s}^{AG}$ . The log-linear model is more parsimonious than the logistic model because it restricts the way in which the intercepts differ across groups by taking the ordinal nature of the response variable into account. As a result, there is only one direct effect parameter in Equation 11 ( $\lambda_{*s}^{AG}$ ) per additional group whereas there are I-1 intercept parameters in Equation 12 ( $\alpha_{is}^{AIG}$ ) per additional group.

Above we showed how to define multigroup LC models for ordinal items by restricting the log-linear and logistic parameters of the model for nominal items using the category scores for the indicators. This amounts to using an adjacent category ordinal logit specification. Alternative ordinal specifications are, among others, cumulative logit and cumulative probit models. Multigroup LC models using such response models could also be specified either with direct effects and interactions or group-specific intercepts and slopes.

### 1.3 Multigroup LC models with ordinal latent variables

LC models with discrete-ordinal latent variables are called LC factor models since they in many ways resemble linear factor analysis (Magidson and Vermunt, 2001; Vermunt and Magidson, 2005). In most aspects multigroup LC factor analysis is equivalent to standard multigroup LC analysis with the main difference being that instead of comparing typologies it compares latent dimensions of observed discrete variables across groups (Moors, 2003; Kankaraš and Moors, 2009).

Let us restrict ourselves to the situation in which there is a single latent variable. The latent variable is modelled as ordinal by using equidistant category scores  $v_t^X$  between 0 and 1 for the levels of the latent variable X in its relationship with the indicators. For example, in the case of a three-level latent variable X ( $t=1, 2,$  and  $3$ ) these scores are:

$$v_t^X = \{0 \text{ if } t=1, 0.5 \text{ if } t=2, 1 \text{ if } t=3\} \quad (13)$$

with the following constrains in the log-linear specification:

$$\lambda_{it}^{AX} = v_t^X \lambda_{i*}^{AX} \quad \text{and} \quad \lambda_{its}^{AXG} = v_t^X \lambda_{i*s}^{AXG} \quad (14)$$

and in the logistic specification:

$$\beta_{its}^{AXIG} = v_t^X \beta_{i*s}^{AXIG} \quad (15)$$

Note that the two parameterizations are equivalent because  $\beta_{i*s}^{AXIG} = \lambda_{i*}^{AX} + \lambda_{i*s}^{AXG}$ .

It is also possible to define both the latent and the indicators to be ordinal, which yields

$$\lambda_{is}^{AG} = v_i^A \lambda_{*s}^{AG}, \lambda_{it}^{AX} = v_i^A v_t^X \lambda_{**}^{AX} \quad \text{and} \quad \lambda_{its}^{AXG} = v_i^A v_t^X \lambda_{**s}^{AXG} \quad (16)$$

and:

$$\beta_{its}^{AXIG} = v_i^A v_t^X \beta_{**s}^{AXIG} \quad (17)$$

Similar to the multigroup LC models with ordinal indicators and a nominal latent variable described in the previous subsection, the loglinear and logistic formulations are not completely equivalent anymore, as direct effects and intercepts contain different numbers of parameters. Although LC factor models are typically used with ordinal indicators, multigroup LC factor models with nominally defined indicators can be very useful in cross-cultural research as it allows for simultaneous analysis of measurement invariance and various response styles that can occur in survey responses (Moors, 2004).

## **2. ANALYZING MEASUREMENT INVARIANCE**

In the multigroup LC models presented in the previous section, all model parameters were assumed to differ across groups, which makes it difficult to compare the results across groups. However, these are not the types of models a cross-cultural researcher is aiming at since he or she wants to be able compare results across groups. To determine whether this is possible, the researcher has to check whether latent classes have the same meaning in all groups, i.e. whether measurement invariance can be established. In the context of LC analysis measurement invariance is established when the class-specific conditional response probabilities are equal across groups. This implies that it is necessary to impose across-group equality restrictions on these conditional probabilities in order to test for measurement equivalence. As is shown below, using a multigroup LC analysis approach, various levels of homogeneity (i.e. measurement invariance) can be tested, each of which involves restricting specific sets of model parameters to be equal across groups.

### **2.1 The general procedure of analysing measurement invariance**

The ideal situation for an applied researcher who wishes to compare groups occurs when all measurement model parameters can be set equal across groups. From this perspective, the objective of researching measurement invariance is to find the model with the lowest level of inequivalence possible that fits the data well. The model selection procedure usually starts by determining the required number of latent classes or discrete latent factors for each group. How this is determined will be explained later on. If the number of classes is the same across groups, then the heterogeneous model is fitted to the data; followed by a series of nested, restricted models which are evaluated in terms of model fit (McCutcheon, 2002, Hagenaaars, 1990; Eid, Langeheine, and Diener 2003).

Graphical representations of the four prototypical models that differ in the assumed level of measurement invariance are provided in Figure 1 and explained in the remainder of this section of the paper. The heterogeneous, unrestricted multigroup LC model, as we have described in the first section of this paper (cf. Equations 1, 3, and 5) is graphically presented in Figure 1a in which X represent the latent variable, M the set of manifest variables, and G is the group variable.

Model 1a represent the situation of complete lack of comparability of results across groups as all measurement model parameters are group specific. Comparability is only established if we can impose across-groups restrictions on the model parameters without deteriorating the fit with the data. Imposing restrictions create various nested *homogeneous* models. If some, but not all, of the model parameters are restricted to be equal across groups; the model is called *partially homogeneous* (Clogg and Goodman, 1984; 1985).

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Insert Figure 1 here

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Among the various possible partially homogeneous models, the one presented in Figure 1b with no ‘group - latent variable’ interaction terms is especially important.

This model implies the following restrictions:

$$\lambda_{its}^{AXG} = \lambda_{jts}^{BXG} = \lambda_{kts}^{CXG} = \lambda_{lts}^{DXG} = 0 \quad (18)$$

or

$$\beta_{its}^{AXIG} = \beta_{it}^{AX}, \text{ etc.} \quad (19)$$

which results in the following equations for the group-specific conditional response probabilities:

$$\pi_{its}^{AlXG} = \frac{\exp(\lambda_i^A + \lambda_{it}^{AX} + \lambda_{is}^{AG})}{\sum_i \exp(\lambda_i^A + \lambda_{it}^{AX} + \lambda_{is}^{AG})}, \text{ etc.} \quad (20)$$

or

$$\pi_{its}^{AlXG} = \frac{\exp(\alpha_{is}^{AlG} + \beta_{it}^{AX})}{\sum_i \exp(\alpha_{is}^{AlG} + \beta_{it}^{AX})}, \text{ etc.} \quad (21)$$

Thus, this model still allows for ‘direct effects’ of the grouping variable on the indicator items ( $\lambda_{is}^{AG}$ ) or in second formulation it allows for group-specific intercept parameters ( $\alpha_{is}^{AlG}$ ). This means that the values of the conditional response probabilities (i.e., their “difficulties”) are different across populations. However, as there are no group – latent variable interaction effects in the model (as slope parameters are assumed to be equal across groups), relationships between the latent variable and the responses are the same across groups, which makes it possible to compare group differences in latent class membership (McCutcheon and Hagenaaars, 1997). It should be noted the partially homogeneous model presented in Equations 20 and 21 can only be specified with loglinear and logistic parameterizations -

distinguishing direct and interaction effects and intercepts and slope parameters, respectively - and thus not with the probabilistic parameterization. This is conceptually similar with the ‘metric equivalence’ model in MCFA in which factor loadings are equal across groups, but item intercepts may be unequal. Likewise, it resembles the situation of ‘uniform’ differential item functioning (DIF) in IRT modelling. The partially homogeneous model can be tested against the unrestricted heterogeneous model. If the difference between the two models is not significant, a researcher can conclude that interaction effects are not needed in the model and can proceed with the next step in the analysis.

In comparative social research, researchers are typically interested in establishing full comparability of the measurement across groups - that is they want to attain complete measurement invariance. In order to do so in the context of LC models it is necessary to establish *structural equivalence* (McCutcheon, 2002). In a *structurally equivalent (homogeneous)* model (Figure 1c) both direct and interaction effects are excluded from the loglinear model (set to zero), or in the alternative logistic formulation both intercept and slope parameters are set to be equal across groups. This means that the conditional probabilities of items are restricted to be equal across groups (e.g.,  $\pi_{it1}^{AXIG} = \pi_{it2}^{AXIG} = \dots = \pi_{itS}^{AXIG}$ ) making the indicator variables independent of the group variable, when controlled for the latent variable. The structurally equivalent LC model then takes the following form:

$$\pi_{ijklts}^{ABCDXIG} = \pi_{ts}^{XIG} \pi_{it}^{AIX} \pi_{jt}^{BIX} \pi_{kt}^{CIX} \pi_{lt}^{DIX} \quad (22)$$

or in loglinear form:

$$\pi_{its}^{AIXG} = \pi_{it}^{AIX} = \frac{\exp(\lambda_i^A + \lambda_{it}^{AX})}{\sum_i \exp(\lambda_i^A + \lambda_{it}^{AX})}, \text{ etc.} \quad (23)$$

and in logistic terms:

$$\pi_{its}^{AlXG} = \pi_{it}^{AlX} = \frac{\exp(\alpha_i^A + \beta_{it}^{AX})}{\sum_i \exp(\alpha_i^A + \beta_{it}^{AX})}, \text{ etc.} \quad (24)$$

Thus, in the structurally equivalent model the relationships between indicator items and the latent variable are identical across groups so that the class memberships have the same meaning in all groups. In other words, measurement invariance is established if this model does not fit the data significantly worse than the partially homogenous and heterogeneous models. The homogeneous model is comparable with the ‘scalar equivalent’ model in MCFA that defines both factor loadings and item intercepts to be the same across groups. In the IRT approach, it is similar to the model with both ‘difficulty’ and ‘discrimination’ parameters invariant across groups.

Finally, if all parameters are restricted to be equal across group – that is if aside from conditional response probabilities, LC probabilities are also independent of group membership ( $\pi_{t1}^{XIG} = \pi_{t2}^{XIG} = \dots = \pi_{tS}^{XIG}$ ), then we have the case of a *completely equivalent (homogeneous)* model (Figure 1d):

$$\pi_{ijklts}^{ABCDXIG} = \pi_{ijt}^{ABX} = \pi_t^X \pi_{it}^{AlX} \pi_{jt}^{BlX} \quad (25)$$

or in the loglinear parameterization:

$$\pi_{ts}^{XIG} = \frac{\exp(\gamma_t^X)}{\sum_t \exp(\gamma_t^X)} \quad (26)$$

For researchers in comparative social research the latter model is of less practical relevance, since the very aim of cross-cultural research is typically to describe country differences in LC membership probabilities or factor means and, hence, to illustrate cross-cultural diversity.

Research is not by definition restricted to comparing the four models drawn in figure 1. Various combinations of within- and across-groups restrictions and different parameterizations are possible. One of these possibilities is, for instance, to test for

equal error rates of the indicator variables by restricting the corresponding conditional probabilities within a group to be equal (McCutcheon and Hagenars, 1997).

The procedure we just explained includes an analysis at the scale level – that is all indicator variables in the model are simultaneously modelled with the same set of restrictions. However, multigroup LC analysis of measurement invariance can be conducted at the item level as well. This is particularly relevant when the scale level analysis indicates inequivalence either in the interaction or in the direct effects. In that case the analysis continues with item level comparisons in order to check whether all items cause inequivalence. More specifically, equivalence in the slope parameter (presence of interaction effect) for a particular item A is assessed by comparing the unrestricted, heterogeneous model with a model in which this parameter is equated across groups for this item. In order to test for equivalence in intercept parameters (presence of direct effects) at the item level we need to assume equivalence in the slope parameters. Therefore, testing equivalence of the intercept parameters of item A is based on the comparison of the partially homogeneous model with equal slope parameters for all items (Equations 20 and 21) with the model which in addition assumes equal intercept parameters for item A. This procedure is very similar to the one used in MCFA where it is referred to as ‘partial equivalence’ (Steenkamp and Baumgartner, 1998; for a discussion on the MCFA approach to partial invariance see also Lee et al., in this book). It should be noted that multiple LC analysis differs from MCFA in that it does not require the use of an invariant marker item for identification purposes.

As we have noted before, the first step in a multigroup LC analysis is to determine whether the number of latent classes or the number of discrete factors is the same across groups (McCutcheon, 1987; McCutcheon and Hagenars, 1997).

However, it might very well be that a model with an acceptable fit in one group has more latent classes than the best fitting model in another group; and in a LC factor model the best fitting model in one country may have more factors than in other countries. In MCFA, the latter situation is referred to a violation the ‘configural invariance’ assumption (Steenkamp and Baumgartner, 1998), which limits the possibility of group comparisons. However, multigroup LC analysis with a nominal latent variable is rather flexible in the sense that it can be used to accommodate different numbers of latent classes across groups while still assuming measurement invariance. This involved specifying a model with same number of classes in each group, but in which some of the classes being empty (having proportions of 0) in certain groups. An example could be a three-class model with class proportions of 0.2, 0.3, and 0.5 for one group and of 0.4, 0.6, and 0.0 for the other group. The analysis of measurement invariance can proceed as described above.

The flexibility of the multigroup LC approach is also reflected in the fact that not all latent classes need to be equivalent in order to validly compare results across groups. In other words, there may be a situation in which only some of the latent classes have the same conditional response probabilities across groups, while other latent classes in a model do not. If this is the case, it is still possible to compare class sizes of equivalent classes while treating other classes in the model as group-specific and non-comparable. Models of this type can be defined using the probabilistic parameterization of the multigroup LC model.

## **2.2 Parameter estimation and assessment of model fit**

LC models are usually estimated by means of maximum-likelihood (ML) under the assumption of a multinomial distribution for the indicator variables in model.

Maximization of the likelihood function is performed by the use of an expectation-maximization (EM) or a Newton-Raphson algorithm, or a combination of these two.

There are several model fit criteria that are commonly used for model fit evaluation in multigroup LC analysis. The likelihood-ratio chi-square ( $L^2$ ) statistic is used as a standard measure of discrepancy between observed and expected frequencies in the model. This statistic has one important advantage over the Pearson chi square ( $X^2$ ) test that lays in its partitioning ability. In particular, when two models are nested and when the less restricted model fits the data well, then the difference in the likelihood ratios between the two models represents a conditional likelihood ratio ( $L^2$ ) test on its own, following a chi-square distribution with a number of degree of freedom equal to the difference between the degrees of freedom of the two nested models. Thus, this conditional likelihood test can be used to compare the fit of successive, nested models and so to investigate the plausibility of (measurement invariance) restrictions included in nested models.

However, the likelihood ratio chi square test, although extensively used in statistical literature, has a number of important limitations. The major one is its limited use when dealing with sparse tables, i.e. when the number of possible response patterns is large and the sample size is small creating contingency tables with many small and zero observed frequencies. In these cases p-values of the chi-square tests can not be trusted as they might not follow the theoretical chi square distribution. On the other hand, when sample sizes are large, likelihood-ratio tests tend to be too conservative, indicating misfit even for minimal differences between two models. In addition, the likelihood-ratio statistic does not provide enough control for the number of parameters in a model that can sometimes be very large even for models of modest size (McCutcheon, 2002).

These limitations prompted the recent development and use of several information criteria, such as the Akaike information criterion (AIC), Bayesian information criterion (BIC), modified AIC (AIC3), and consistent AIC (CAIC), each of which is designed to penalize models with larger numbers of parameters. Since more parameters in a model increase its likelihood, the information criteria reduce that likelihood by a certain amount that is a function of the increased number of estimated parameters. They differ in the specific function with which they calculate the penalizing value for each additional parameter in a model. Specifically, AIC and AIC3 rely solely on the number of parameters in the model:

$$\text{AIC} = L^2 - 2df \quad \text{and} \quad \text{AIC3} = L^2 - 3df \quad (27)$$

while BIC and CAIC also takes into account the sample size:

$$\text{BIC} = L^2 - df*[\ln(N)] \quad \text{and} \quad \text{CAIC} = L^2 - df*[\ln(N)+1] \quad (28)$$

where N is sample size. Thus, models with lower values of information criteria have a better fit to a data, for a given number of parameters. Since they also control for sample size, BIC and CAIC are preferred fit statistics in situations when sample size is large. For small to medium sample sizes, the AIC statistic is most commonly used.

Software packages that can be used to obtain ML estimates and model fit statistics of the LC models are LEM (Vermunt, 1997), Latent GOLD (Vermunt and Magidson, 2005, 2008), MPlus (Muthén and Muthén, 2006), and GLLAMM (Rabe-Hesketh, Skrondal and Pickles, 2004).

### 3. EMPIRICAL EXAMPLES

In this section we will present two examples of the use of LC models for the analysis of measurement invariance. The first example involves a standard multigroup LC analysis as both latent and indicator variables are treated as nominal. Multigroup

LC factor analysis is illustrated in the second example with latent and indicator variables defined to be discrete ordinal. In both examples we used equal size weighting of the samples to a sample size of 1000 per country, which was the size of the smallest country sample. This procedure is often used in cross-cultural research to prevent countries with larger sample sizes to dominate the results.

The analyses of measurement invariance follow the procedures outlined in the section 2.1, by first selecting the best fitting model at the scale level and then testing invariance of individual items at the item level. We use the BIC statistic as our main model selection fit criterion since both the conditional  $L^2$  test and AIC do not provide control mechanism for sample size and are thus too conservative in their model fit evaluation with sample sizes as large as the ones used in presented examples. Models were estimated with the syntax version of the Latent Gold 4.5 program (Vermunt and Magidson, 2008). The syntax used in the two examples is reported in the Appendix.

### **3.1 Example 1: Standard multigroup LC analysis**

The first example involves the analysis of four categorical items on preferences with respect to social developments in different spheres of life taken from the 1999/2000 European Value Survey. Respondents were asked whether it would be a good thing, a bad thing, or whether they didn't mind if in the near future emphasis would be placed on the development of 1) technology, 2) the individual, 3) family life, and 4) natural lifestyle. Though it is hard to imagine that these four issues refer to a single dimension, it can be that groups of respondents can be identified with different preferences, which is why a traditional LC analysis approach is used. Models are defined using the logistic parameterisation presented in Equation 5. We compare results between four countries: Belarus, Romania, Luxembourg, and Austria. These

countries were chosen because this particular selection allowed us to demonstrate a number of important features of the method.

Separate analyses for each country indicated that a 2-class model provides the best fit in terms of BIC statistics. Whether a common 2-class structure emerges from the data, i.e. whether the data are measurement invariant, can be tested by fitting a 2-class model with the pooled dataset. The level of measurement invariance present in data is indicated by the degree of homogeneity in the model that fits the data best. The more homogeneous the best-fitting model is, the more equivalent the data are.

In Table 1a, we report the fit statistics for the various multigroup 2-class models that were estimated. As can be seen, the partially homogeneous model with equal loadings but different intercept parameters across countries (depicted in Figure 1b), fits the data best (BIC=-1758,4). This indicates that the estimated class-specific response probabilities for the two classes are not exactly the same across countries.

There is a second route to be explored. It is possible that some latent classes can be observed in all countries, whereas other latent classes are country specific. In that case, a 2-class model for the pooled dataset would not be the best choice, but instead a model with more classes would be better. Hence, an alternative way to investigate the source of invariance is by checking whether the inclusion of (an) additional class(es) improves the model fit. In Table 1b, we report the fit statistics of the same three multigroup LC models but now with 3 instead of 2 classes. As we can see, the best fitting 3-class model is the structurally homogeneous (measurement invariant) model. Obviously, the addition of the third class has accounted for a substantial part of the inequivalence encountered in the 2-class model. This indicates that the partial inequivalence found in the 2-class model can, at least partially, be explained by

selecting a model with too few classes. Once the third class is included in the model, the latent classes turn out to be equivalent and comparable.

Whereas the analyses presented so far were at the scale level, it is also useful to perform an item-level analysis to check the invariance of individual items. It should be some items may turn out to be non-invariant even if the scale-level analysis selects the homogeneous 3-class model. In sections c and d of Table 1, we present the fit measures obtained with the item-level analysis for the four items, both in terms of absence of interaction effects or invariance in slope parameters (Table 1c) and in terms of absence of direct effects or invariance in intercept parameters (Table 1d). As could be expected, all items have invariant slopes as BIC values of models without interaction effects for one item at a time ( $H_{4a} - H_{4d}$ ) are smaller than that of the heterogeneous model  $H_4$ . However, one of the items, i.e. 'assessing the preferred development of the individual', turns out to be inequivalent in terms of its intercept as is indicated by a higher BIC for model  $H_{5b}$  compared to the partially homogeneous model  $H_5$ . In other words, respondents' differences in answering this question were not only determined by membership to given latent classes but also by additional group-specific factor(s) that are unrelated to class membership. Therefore, in order to validly compare the class proportions across countries we will need to include the direct effect of countries on this indicator in the measurement model – that is, to allow the intercept of this indicator to vary across countries. Thus, the final measurement model is the model  $H_7$  (Table 1e) which is equal to the structural homogeneous model  $H_6$  modified by adding the direct effect of the grouping variable 'country' on the item 'individual'. All other parameters in the model are invariant across countries.

---

Insert Table 1 here

---

Having selected a measurement model that allows for comparison of countries, the next two questions refer to: (a) a substantive interpretation of the latent classes, and (b) the comparison of class sizes across countries. In Table 2 we report the item response probabilities and class proportions obtained for the selected  $H_7$  model.

---

Insert Table 2 here

---

Class 1 comprise 58,8% of respondents with overwhelmingly positive preference towards all four social developments; 32,9% of people belonging to class 2 have somewhat more negative and less involved preferences towards development of technology; 8,3% of respondents belonging to class 3 are rather indifferent in respect to the given subject (have high percentage 'don't mind' answers).

Class sizes differ substantially across countries. Most of the respondents in Belarus and Romania belong to class 1 and have positive preferences for all social developments, whereas in Luxembourg and Austria there is also a considerable number of people belonging to class 2 with more reserved views on development of technology. The third class containing the less concerned respondents is smallest in all four countries. To test whether class sizes differ significantly across countries we compare the selected model  $H_7$  with a model in which equal class sizes are assumed (model  $H_{7a}$ ). Fit statistics of this model ( $H_{7a}$ ) presented in Table 1e show that it fits much worse than model  $H_7$ , which indicates that the obtained differences in class sizes across countries are statistically significant.

### **3.2 Example 2: Multigroup LC factor analysis**

In this second example we illustrate multigroup LC factor analysis with an application to a set of discrete ordinal indicators from the 2006/2007 European Social Survey (ESS), which contains information on 23 European countries. The records were weighted in order to yield an equal number of 1000 cases per country. We investigated the measurement invariance of a 4-item scale measuring inter-personal feelings which assesses to what extent respondents a) feel that people in their local area help one another, b) feel that people treat them with respect, c) feel that people treat them unfairly, and d) feel that they get the recognition they deserve. Answers are given on a 7-point rating scale ranging from ‘Not at all’ to ‘A great deal’. We modelled the indicators and the latent variable (3-levels) as discrete-ordinal, using the logistic parameterization of the LC factor model presented in Equation 17.

In Table 3, we report the likelihood ratio ( $L^2$ ), BIC and AIC statistics for various LC factor models. On the scale level (Table 3a), we compared three basic LC models: the heterogeneous model  $H_1$ , the partially homogeneous model  $H_2$  without interaction effects between the latent and grouping variable (with equal slope parameters), and the measurement invariant, homogeneous model  $H_3$  with neither direct nor interaction effects (with equal intercept and slope parameters). As we see, the BIC statistic indicates that the homogeneous  $H_3$  model fits the data best taken into account the number of parameters and the sample size. However, before drawing a final conclusion about measurement invariance, we need to check whether all individual items are measurement invariant.

---

Insert Table 3 here

---

In the item-level analysis, we first compared the heterogeneous model  $H_1$  with four models ( $H_{1a} - H_{1d}$ ) in which the interaction effect between the latent and grouping variable are excluded for one item at a time (Table 3b). Since the four models excluding a single interaction term do not fit worse than the unrestricted model  $H_1$ , we can conclude that there are no significant interaction effects and the relationship between the latent variable and indicators can be assumed to be the same across countries, which confirms what we found in the scale-level analysis.

The next step involves testing of the need for direct effects at the item level comparison the four models  $H_{2a} - H_{2d}$  that excludes the direct effects of the grouping variable on a single item with the partially homogeneous  $H_2$  (Table 3c). The fit measures show that none of these restricted models fits worse than the partially homogeneous model  $H_2$ , which indicates that the conditional response probabilities can be assumed to be equal across countries for each of the four items. Thus, our analysis shows that the scale designed to measure inter-personal feelings is measurement invariant. This means that the four indicator items are measuring one latent variable in all of the 23 countries and that the meaning of this latent variable is the same across countries. Having established measurement invariance, a researcher can now proceed with the analysis of substantive differences in latent variable across countries.

Class proportions and discrete factor means for each country are reported in Table 4. The latter are calculated by multiplying class proportions with predefined fixed scores 0, 0.5, and 1 of each factor level. The level of positive feelings increases with class number.

---

Insert Table 4 here

---

The estimates reported in Table 4 indicate that respondents from Denmark, Norway and Switzerland have the most positive, and from Slovakia, Ukraine, and United Kingdom the most negative feelings about their relationship with other people.

## CONCLUSIONS

The paper discussed the use of multigroup LC analysis as a tool for investigating measurement invariance. Three parameterizations of the multigroup LC models, were presented, i.e. a probabilistic, log-linear and logistic parameterization. The latter two are used to define the LC model with ordinal indicator variables and the LC factor model. An additional benefit of the log-linear and logistic parameterization is that they are better suited for testing measurement invariance, as they allow a researcher to test a whole range of partially homogeneous models that are not possible to formulate using probabilistic parameterization. It was shown how to test for strict and less strict forms of measurement invariance by gradually imposing restrictions on the fully heterogeneous unrestricted multigroup LC model and comparing the resulting model fit statistics.

The LC approach is an obvious choice when a researcher wishes to compare typological structures across countries – that is when analyzing whether there are cross-cultural differences in the frequencies of the different types, taking into account issues of measurement equivalence. With the possibility to define the latent variable as discrete-ordinal, it is shown that the LC approach can also be used for cross-cultural comparisons of dimensional structures, thus, presenting an alternative to the more frequently used MCFA and IRT approaches. This is especially true in those situations when some of the modeling assumptions of MCFA and IRT do not hold.

With its flexible set of tools, combined with recent developments in software for multigroup LC modelling, the presented approach is a very attractive option for studying measurement invariance in any situation in which the indicators are discrete variables.

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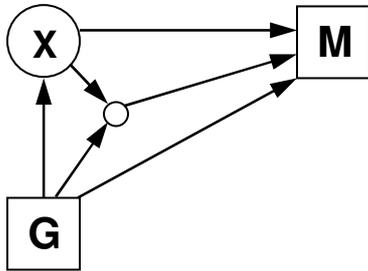
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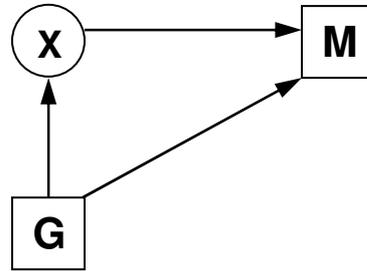
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## FIGURES AND TABLES

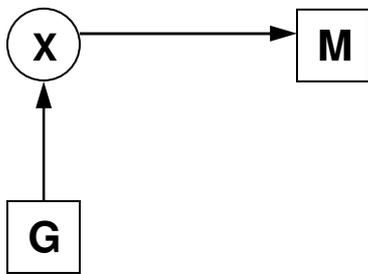
**Figure 1** Relationships between latent variable (X), manifest variables (M) and group variable (G) in four different multigroup LC models



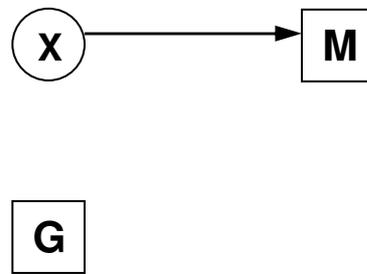
a) Heterogeneity (complete inequivalence)



b) Partial homogeneity



c) Structural homogeneity



d) Complete homogeneity

**Table 1:** Fit statistics of the estimated 2- and 3-class multigroup LC models

	<b>Npar*</b>	<b>L<sup>2</sup></b>	<b>BIC(L<sup>2</sup>)</b>	<b>df</b>
<b>a) H<sub>1</sub>: Heterogeneous 2-class</b>	<b>68</b>	<b>402,5</b>	<b>-1642,4</b>	<b>252</b>
H <sub>2</sub> : Partial Homogeneity (Figure 1b)	44	481,2	-1758,4	276
H <sub>3</sub> : Structural Homogeneity	20	822,7	-1611,7	300
<b>b) H<sub>4</sub>: Heterogeneous 3-class</b>	<b>104</b>	<b>241,5</b>	<b>-1511,2</b>	<b>216</b>
H <sub>5</sub> : Partial Homogeneity (Figure 1b)	56	352,6	-1789,6	264
H <sub>6</sub> : Structural Homogeneity	32	512,4	-1824,6	288
<b>c) H<sub>4</sub>: Heterogeneous 3-class</b>	<b>104</b>	<b>241,5</b>	<b>-1511,2</b>	<b>216</b>
H <sub>4a</sub> : Technology	92	276,8	-1573,4	228
H <sub>4b</sub> : Individual	92	265,4	-1584,7	228
H <sub>4c</sub> : Family	92	274,2	-1575,9	228
H <sub>4d</sub> : Natural lifestyle	92	283,3	-1566,8	228
<b>d) H<sub>5</sub>: Partial homogeneity 3-class</b>	<b>56</b>	<b>352,6</b>	<b>-1789,6</b>	<b>264</b>
H <sub>5a</sub> : Technology	50	395,7	-1795,3	270
H <sub>5b</sub> : Individual	50	409,1	-1781,9	270
H <sub>5c</sub> : Family	50	373,6	-1817,3	270
H <sub>5d</sub> : Natural lifestyle	50	365,1	-1825,9	270
<b>e) H<sub>7</sub>: Selected 3-class</b>				
H <sub>7</sub> : H <sub>6</sub> with 1 direct effect	<b>38</b>	<b>439,3</b>	<b>-1849,1</b>	<b>282</b>
H <sub>7a</sub> : H <sub>7</sub> with equal class sizes	32	729,3	-1607,8	288

\* Number of parameters for heterogeneous models with nominal indicators is calculated in following way:  $Npar = (A-1) + [(A-1) \times (B-1)] + B \times [C + D]$ , with  $C = (E-1) \times F$  and  $D = (A-1) \times (E-1) \times F$ , where A is number of clusters; B is number of items; C is number of intercept parameters; D is number of loadings parameters; E is number of response categories; and F is number of countries. For partially homogeneous models D changes to:  $D = (A-1) \times (E-1)$ ; for structurally homogeneous models C additionally changes to:  $C = E-1$ .

**Table 2:** Item response and class probabilities for preferences of social development

<b>a. Response probabilities</b>	<b>Class 1</b>	<b>Class 2</b>	<b>Class 3</b>
<b>Technology</b>			
Good	0,947	0,369	0,541
Bad	0,023	0,312	0,027
Don't mind	0,030	0,319	0,432
<b>Individual (average across countries)</b>			
Good	0,985	0,734	0,488
Bad	0,008	0,089	0,032
Don't mind	0,006	0,176	0,480
<b>Family</b>			
Good	0,972	0,917	0,506
Bad	0,003	0,041	0,025
Don't mind	0,025	0,042	0,469
<b>Natural lifestyle</b>			
Good	0,884	0,868	0,235
Bad	0,048	0,073	0,096
Don't mind	0,068	0,059	0,669
<b>b. Latent class proportions</b>			
Belarus	0,753	0,134	0,114
Romania	0,843	0,103	0,054
Luxembourg	0,467	0,458	0,075
Austria	0,336	0,578	0,086
<b>Total</b>	<b>0,588</b>	<b>0,329</b>	<b>0,083</b>

**Table 3** Fit statistics for the estimated multigroup LC factor models

<b>Model</b>	<b>Npar</b>	<b>L<sup>2</sup></b>	<b>BIC(L<sup>2</sup>)</b>	<b>df</b>
<b>a) H<sub>1</sub>: Heterogeneous model</b>	<b>668</b>	<b>27511,0</b>	<b>-179865,0</b>	<b>20792</b>
H <sub>2</sub> : Partial homogeneity (Figure 1b)	580	28146,1	-180107,6	20880
H <sub>3</sub> : Structural homogeneity	52	32336,1	-181183,8	21408
<b>b) H<sub>1</sub>: Heterogeneous model</b>	<b>668</b>	<b>27511,0</b>	<b>-179865,0</b>	<b>20792</b>
H <sub>1a</sub> : Item 1	646	27698,6	-179896,7	20814
H <sub>1b</sub> : Item 2	646	27559,8	-180035,6	20814
H <sub>1c</sub> : Item 3	646	27691,9	-179903,5	20814
H <sub>1d</sub> : Item 4	646	27674,6	-179920,7	20814
<b>c) H<sub>2</sub>: Partial homogeneity</b>	<b>580</b>	<b>28146,1</b>	<b>-180107,6</b>	<b>20880</b>
H <sub>2a</sub> : Item 1	448	28961,3	-180608,9	21012
H <sub>2b</sub> : Item 2	448	29362,1	-180208,2	21012
H <sub>2c</sub> : Item 3	448	29114,0	-180456,3	21012
H <sub>2d</sub> : Item 4	448	29228,4	-180341,8	21012

**Table 4** Latent class proportions and latent means for the 23 countries

Country	Proportions			Means
	Class 1	Class 2	Class 3	
Austria	0,314	0,538	0,147	0,417
Belgium	0,314	0,578	0,108	0,397
Bulgaria	0,388	0,441	0,169	0,391
Switzerland	0,134	0,625	0,240	0,553
Cyprus	0,296	0,534	0,169	0,436
Germany	0,321	0,566	0,112	0,395
Denmark	0,088	0,641	0,270	0,591
Estonia	0,345	0,518	0,136	0,395
Spain	0,192	0,586	0,221	0,515
Finland	0,272	0,603	0,123	0,426
France	0,313	0,552	0,134	0,410
United Kingdom	0,419	0,487	0,093	0,337
Hungary	0,219	0,462	0,318	0,550
Ireland	0,215	0,520	0,263	0,524
Netherlands	0,245	0,608	0,145	0,450
Norway	0,106	0,639	0,253	0,574
Poland	0,406	0,495	0,097	0,345
Portugal	0,221	0,571	0,207	0,493
Russian Federation	0,387	0,499	0,113	0,363
Sweden	0,140	0,655	0,203	0,532
Slovenia	0,350	0,512	0,137	0,393
Slovakia	0,550	0,392	0,057	0,253
Ukraine	0,466	0,433	0,100	0,317
<b>Total</b>	<b>0,290</b>	<b>0,543</b>	<b>0,166</b>	<b>0,437</b>

### APPENDIX – Latent Gold syntax files used in the two examples

This appendix we present the “variables” and “equations” sections of the Latent GOLD 4.5 syntax files used for the two examples in this chapter. The logistic parameterization of the heterogeneous multigroup LC model with a 3-class nominal latent variable and 4 nominal indicator variables as used in example 1 is as follows:

```
variables
  caseweight weight;
  dependent item1 nominal, item2 nominal, item3 nominal,
    item4 nominal;
  independent country nominal;
  latent Cluster nominal 3;

equations
  Cluster <- 1 | country;
  item1 <- 1 | country + Cluster | country;
  item2 <- 1 | country + Cluster | country;
  item3 <- 1 | country + Cluster | country;
  item4 <- 1 | country + Cluster | country;
```

In the “variables” section, one provides the relevant information on the dependent (items), independent (here the grouping variable), and latent (here the latent classes) variables to be used in the analysis. In this analysis, these are all nominal variables, where for the latent variable called “Cluster” one also has to indicate how many categories it has.

The first “equation” defines the logistic model for the class proportions (“1” indicates the intercept), which are assumed to be different across countries (indicated with “| country”). The next four equations define the logistic regression models for the four items. These contain the term “1” referring to the intercept and the term “Cluster” referring to the slope. Both are indicated to differ across countries.

Other, more restricted models are obtained with slight modifications of the equations. A model assuming invariant item intercepts and/or slopes across countries is obtained by removing “| country” from the term(s) concerned. Thus, a partially homogeneous model is defined by equations of the form:

```
item# <- 1 | country + Cluster;
```

and the homogeneous model by:

```
item# <- 1 + Cluster;
```

A log-linear parameterization of these models can be defined by writing “+ country” instead of “| country” for intercepts and “+ Cluster country” instead of “| country” for slope parameters. The item equations of the heterogeneous model would then be as follows:

```
item# <- 1 + country + Cluster + Cluster country;
```

Finally, the only modification needed to obtain a multigroup LC factor model for ordinal items (our second example) is that the dependent and latent variables should be defined to be ordinal instead of nominal:

```
dependent item1 ordinal, item2 ordinal, item3 ordinal,  
item4 ordinal;  
latent factor ordinal 3;
```

The “equations” remains the exactly the same as with nominal dependent and latent variables, though it should be noted that the log-linear and logistic parameterizations are no longer equivalent with ordinal indicators.