Non-parametric Random-Effects Model

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RANDOM-EFFECTS MODELING is one of the several alternative approaches to deal with dependent observations such as that which occur in repeated measures or multilevel data structures. Non-parametric random-effects models differ from standard (parametric) random-effects models in that no assumptions are made about the distribution of the random effects. Actually, this is a form of LATENT CLASS ANALYSIS: the mixing distribution is modelled by means of a finite mixture structure. Early references to the non-parametric approach are Laird (1978) and Heckman and Singer (1982).

Let y_{ij} denote the response variable of interest, where the index j refers to a group and i to a replication within a group. Note that with repeated measures, groups and replications refer to individuals and time points. It is easiest to explain the random-effects models within a GENERALIZED LINEAR MODELING framework; that is, to assume that the response variable comes from a distribution belonging to the exponential family and that the expectation of y_{ij}, $E(y_{ij})$, is modelled via a linear function after an appropriate transformation $g[..]$.

A simple random-intercept model without predictors has the following form:

$$g\left[E(y_{ij})\right] = \alpha_j.$$

To utilize a parametric approach, a distributional form is specified for α_j, typically normal: $\alpha_j \sim N(\mu, \tau^2)$. The unknown parameters to be estimated are the mean μ and the variance τ^2. An equivalent parameterization is $g\left[E(y_{ij})\right] = \mu + \tau \cdot u_j$, with $u_j \sim N(0, 1)$.

A non-parametric approach characterizes the distribution of α_j by an unspecified discrete mixing distribution with C nodes (latent classes), where a particular latent class LC is enumerated by x, $x = 1, 2, ..., C$. α_x The intercept associated with class x is denoted by α_x and the size of class x by $P(x)$. The α_x and $P(x)$ are sometimes referred to as the location and weight of node x. The non-parametric model can be specified as follows:

$$g\left[E(y_{ij}|x)\right] = \alpha_x.$$

The non-parametric maximum likelihood estimator is obtained by increasing the number of latent classes till a saturation point is reached. In practice, however, researchers prefer solutions with less than the maximum number of classes.

The similarity between the pararametric and non-parametric approaches becomes clear when one realizes that the α_x and $P(x)$ parameters can be used to compute the mean (μ) and the variance (τ^2) of the random effects, which are the unknown parameters in the parametric approach. Using elementary statistics, we get $\mu = \sum_{x=1}^{C} \alpha_x P(x)$, and $\tau^2 = \sum_{x=1}^{C} (\alpha_x - \mu)^2 P(x)$.

A more general model is obtained by including predictors z_{ijk}, yielding a two-level regression model with a random intercept:

$$g\left[E(y_{ij}|\mathbf{z}_{ij},,x)\right] = \alpha_x + \sum_{k=1}^{K} \beta_k \cdot z_{ijk}.$$

A special case is the (semi-parametric) Rasch model, which is obtained by adding a dummy predictor for each replication i.

Also the regression coefficient can be allowed to differ across latent classes, which is analogous to having random slopes in a MULTILEVEL ANALYSIS. This yields

$$g\left[E(y_{ij}|\mathbf{z}_{ij},x)\right] = \alpha_x + \sum_{k=1}^{K} \beta_{kx} \cdot z_{ijk},$$

a model that is often referred to as latent class (LC) or mixture regression model. In fact, this is one of the most important applications of latent class analysis: unobserved subgroups are identified which differ with respect to the parameters of the regression model of interest.

Two computer programs for estimating LC regression models are GLIMMIX and Latent GOLD.

References

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