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Latent class regression analysis for describing cognitive developmental phenomena: An application to transitive reasoning

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The aim of cognitive developmental research is to explain latent cognitive processes or structures by means of manifest variables such as age, cognitive behaviour, and environmental influences. In this paper the usefulness of the latent class regression model is discussed for studying cognitive developmental phenomena. Using this model, the relationships between latent and manifest variables can be explained by means of empirical data without the need of strong a priori assumptions made by a cognitive developmental theory. In the latent class regression model a number of classes are distinguished which may be characterized by particular cognitive behaviour. Environmental influences on cognitive behaviour may vary for different (developmental) classes. An application is given of the latent class regression model to transitive reasoning data. The results showed that a Five-Class model best fitted the data and that the latent classes differ with respect to age, strategy use (cognitive behaviour) and the influence of task characteristics (environmental influences) on the strategy use. The flexibility of the model in terms of mixed measurement levels and treatment of different cognitive variables offers a broad application to several cognitive developmental phenomena.

The general aim of cognitive developmental research is the uncovering of relationships between cognitive processes, environmental influences and age (see, e.g., Flavell, 1985; Siegler, 1991). Because cognitive processes can not be observed directly but only inferred from observable variables, observable cognitive behaviour is assumed to indicate the latent cognitive processes. In Figure 1, a general model is displayed of the relationships between observed and latent variables in the domain of cognitive development. The definition and operationalization of the different aspects and relationships in Figure 1 varies for different cognitive developmental theories and the assumptions

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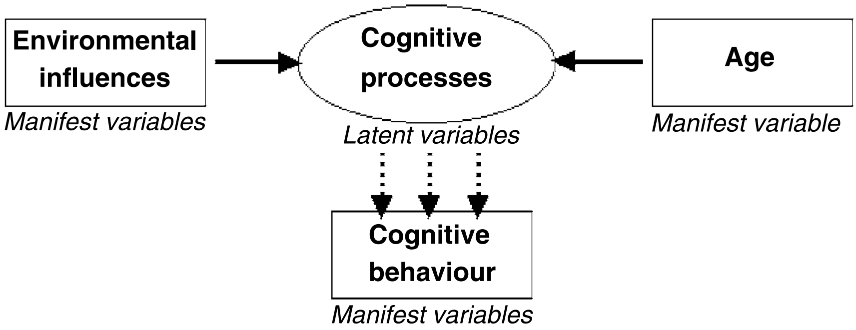

Figure 1. A general model for the relationships between manifest and latent variables in the domain of cognitive development.

about the acquisition of knowledge. Moreover, cognitive developmental theories have different perspectives on the importance of the aspects (Figure 1) and how they should be measured.

For example, in the theory of Piaget (see, e.g., Flavell, 1963; Chapman, 1988; Bidell & Fischer,1992), cognitive abilities are assumed to develop in stages, which are characterized by a particular kind of knowledge structures. One of the most important purposes of Piaget was to give a broad description of the developing structures. Therefore, his theory was domain-general without paying much attention to the influence of external conditions (Case, 1992). In information processing theory (see, e.g., Kail & Bisanz, 1992), however, development is defined as cumulative learning without qualitative change. External experiences make it possible to acquire knowledge, that is, to learn and develop cognitively.

Dependent on the theoretical perspective, assumptions are made about the unobservable (latent) processes and how these processes should be measured using observable variables. Given the assumptions, relationships between observable variables such as age, task conditions and cognitive behaviour, and unobservable variables, such as cognitive processes, are modelled. By studying the observable variables empirically, or by means of computer simulation, one wants to reveal the latent cognitive processes and the relationships between these cognitive processes, environmental influences and age.

However, it is difficult to test a model empirically in which both the observed and the unobserved variables are represented, that is, to estimate and test relationships between observable and unobservable variables without the need for strong cognitive theoretical assumptions. Nevertheless, statistical models in which latent variables can be defined using manifest variables do exist and can be used to study relationships between age, environmental influences and cognitive processes (Embretson, 1985, 1991; Fischer, 1995; Kelderman & Rijkes, 1994; Mislevy & Verhelst, 1990; Sijtsma & Verweij, 1999).

In modern test theory, for example, the observed responses to a number of tasks (e.g., arithmetic problems), which measure a particular ability (e.g., arithmetic ability) are used to determine the number of latent abilities needed to explain the observable data structure, and the strength of the relationships between the item scores and these latent abilities. Thus, modern test models, also known as item response theory (IRT) models (see, e.g., Hambleton & Swaminathan, 1985; Sijtsma & Molenaar, 2002), make it possible to reveal and statistically test a latent structure for explaining the data without the need to posit an a priori theoretical structure stipulated by cognitive theory.

In IRT models the latent variable is continuous, whereas latent class models (e.g., Hagenaars & McCutcheon, 2002) assume latent abilities to be discrete, consisting of two or more nominal or ordered classes. In particular when studying cognitive development, these latent class models are useful to distinguish groups of children on a developmental scale which are characterized by a pattern of specific cognitive behaviour. The cognitive behaviour in a specific latent class may differ, in a particular aspect, from the cognitive behaviour in other latent classes. Latent class models allow the estimation of the classes of the latent variable from the data instead of assuming them on the basis of a cognitive theory. However, latent class models can also be used in a confirmatory way by testing the latent class structure assumed by a cognitive theory.

In the domain of cognitive developmental theory, age is hypothesized to have an important influence on the formation of the latent classes. One may expect that a particular latent class, which is characterized by specific cognitive behaviour, may fit better for children of a particular age range than for children outside this range. Latent class analysis makes it possible to empirically determine the influence of age (as a covariate) on the formation of the latent classes.

A division into classes does not necessarily imply a cognitive stage theory. In contrast, the cognitive behaviour typical of a latent class may be an expression of the same underlying ability continuum. The classes may be ordered and it depends on the level of description of the observed variables whether the interpretation of the latent classes differs quantitatively or qualitatively. For example, Bouwmeester and Sijtsma (2003a) found that the response patterns of children on a set of transitive reasoning tasks could be explained by one ability, but that a broad variety of explanations were used to motivate the responses. Possibly, on a more detailed level, the transitive reasoning ability can be divided into a number of classes that are characterized by a specific pattern of cognitive behaviour.

The power of the latent class model is that specific behaviour patterns can be distinguished and the influence of age determined without a priori cognitive theoretical assumptions. However, it is possible to test a cognitive stage theory using latent class models. Jansen and van der Maas (1997) used a latent class model to empirically study the different stages of reasoning on the balance scale task (Inhelder & Piaget, 1958; Siegler, 1976) and found that the theoretical stages were, together with some others classes, represented by the latent classes.

An additional possibility of latent class models is to describe the classes in more detail by assessing the influence of certain external conditions on cognitive behaviour in a particular class, and compare classes with respect to the influence of external conditions on cognitive behaviour in a set of classes. For this purpose, we used a latent class regression model (Wedel & DeSarbo, 1994; Vermunt & Magidson, 2000), in which a multiple regression function is estimated for a number of classes. The formation of the latent classes is influenced by the covariate age. For every latent class, the influence of external conditions on the cognitive behaviour can be determined. This latent class regression model is a very general and flexible model that can be applied to a broad range of cognitive developmental phenomena. Examples are the development of reasoning on the balance scale task (see, e.g., Jansen & van der Maas, 1997), transitive reasoning (see, e.g., Verweij, 1994), inductive reasoning (see, e.g., De Koning, 2000), and analogical reasoning (see, e.g., Hosenfield, 2003).

The covariate, the dependent variable and the predictor variables can have different measurement levels. For example, instead of age in months, grade level can be used as a covariate, or other child characteristics such as gender, cultural background, or socioeconomic status. Cognitive behaviour may be operationalized as correct/incorrect responses, strategy information, verbal explanations, or reaction times. Predictors may be all kinds of external conditions. For example, tasks may vary in specific task characteristics, or the experiment may take place in different locations or at different times. In the next section an application of the latent class regression model is given in the context of transitive reasoning development.

AN APPLICATION: TRANSITIVE REASONING

In a transitive relation, the relationship, R, between two elements, A and C, can be inferred from their known relationships with a third element, B; that is $(R_{AB}, R_{BC}) \Rightarrow R_{AC}$. In this example, the relationships R_{AB} and R_{BC} are premises. In the research on transitive reasoning a number of different task characteristics are used to study the ability of transitive reasoning. Different kinds of transitive and non-transitive strategies appeared to be used to draw transitive inferences in tasks having different task characteristics (Perner & Mansbridge, 1983; Verweij, 1994; Bouwmeester & Sijtsma, 2003b). Throughout the last decades a discussion has been taking place about which kinds of cognitive behaviour are really expressions of transitive reasoning; which kinds of tasks should be used to measure transitive reasoning; and what really develops when studying transitive reasoning (see, e.g., Smedslund, 1969; Trabasso, 1977; Brainerd & Reyna, 1992; Chapman & Lindenberger, 1992). Therefore, it is important to reveal the relationships between age, cognitive behaviour, and external conditions, when studying the development of this cognitive developmental phenomenon.

METHOD

Instruments

Bouwmeester and Sijtsma (2003a, 2003b) investigated transitive reasoning by constructing a computer test containing 16 transitive reasoning tasks. The tasks differed on three important external conditions, called task characteristics. The task characteristics had 4, 2, and 2 levels defining $4 \times 2 \times 2 = 16$ tasks. A description of the task characteristics is given in Table 1.

Strategies

For each task both the correct/incorrect responses and the verbal explanations were recorded. The verbal explanations associated with the correct/incorrect responses showed that children used a broad variety of

TABLE 1
Description of the transitive reasoning task characteristics

Characteristic	Level	Description
Format	1. $Y_A > Y_B > Y_C$ 2. $Y_A = Y_B = Y_C = Y_D$ 3. $Y_A > Y_B > Y_C > Y_D > Y_E$ 4. $Y_A = Y_B > Y_C = Y_D$	Defines the logical relationships between the objects involved, e.g., when the relationship is length, $Y_A > Y_B > Y_C$ means that object A is longer than object B, which is longer than object C.
Presentation Form	1. Simultaneous 2. Successive	Determines whether all objects are presented simultaneously, or in pairs of two objects during premise presentation.
Content of Relation	1. Physical 2. Verbal	Determines whether the relationship can be perceived visually, or has to be told in words by the experimenter.

TABLE 2
Description of the seven strategies used to solve the transitive reasoning tasks

Name	*Description*	*Example*
1. Literal	All necessary premise information is used to explain the transitive relation.	Object A is longer than object C, because object A is longer than object B and object B is longer than object C.
2. Reduced	The premise information is used in a reduced form.	Animals get older to the right, so the horse is older than the cow because it is positioned before the cow.
3. Incorrect	Premise information is incorrectly used or incorrect premise information is used.	The lion is older than the camel because the hippo and the lion have the same age.
4. Incomplete	Premise information is used correctly but incompletely.	The blue stick is longer than the red stick because the blue stick is longer than the green stick.
5. False memory	The test pair is confused with a premise pair.	I've just seen that the blue stick is longer than the red stick, so that will still be the case.
6. External & visual	Visual or external information is used to explain the transitive relation, no premise information is used.	The parrot is older than the duck because parrots can become very old; When I look very carefully, I can see that the blue stick is longer than the red stick.
7. No explanation	No explanation is given.	I guessed, I just don't know.

explanations but that this differentiation could not be discovered by considering only the correct/incorrect responses. Moreover, Bouwmeester and Sijtsma (2003b) showed that correct/incorrect responses to the tasks of the transitive reasoning test did not form one reliable ability scale. Thus, we used the verbal explanations data in this study. These verbal explanations were categorized into seven strategies, which are displayed in Table 2. These strategies formed the cognitive behaviour investigated by Bouwmeester and Sijtsma (2003a).

Sample

The sample consisted of 615 children stemming from grade two through grade six. Children came from six elementary schools in The Netherlands. They were from middle-class socioeconomic status (SES) families. Table 3 gives an overview of the number of children per grade, and the mean age and the standard deviation of age within each grade.

TABLE 3
Number of children, mean age (M) and standard deviation (SD) per grade

		Age	
Grade	*Number*	*M*[a]	*SD*
2	108	95.48	7.81
3	119	108.48	5.53
4	122	119.13	5.37
5	143	132.81	5.17
6	123	144.95	5.34

[a] number of months.

Data

A representation of the input data file for the latent class regression analysis is shown in Table 4. Each of the 615 children performed 16 tasks (in the table indicated as replications). Each task was defined by a combination of three task characteristics. For example, task 1 had format $Y_A > Y_B > Y_C$, *simultaneous* presentation form, and *verbal* type of content. Each child used one of the seven strategies, and the same child could use different strategies for different tasks.

ANALYSIS: THE LATENT CLASS REGRESSION MODEL

Parts of the model

It was expected that the strategy responses of the children on the 16 transitive reasoning tasks could be divided into a number of classes that were ordered along a developmental scale and differed with respect to specific strategy use for different kinds of transitive reasoning tasks. The formation of the latent classes was expected to be influenced by age. Grade level was used as a covariate instead of age because we were primarily interested in the relationships between grade level and latent class.

The first part of the latent class regression model is defined by the probability (π) of being in a particular latent class (realization x of latent variable X), given grade level (realization z^c, of covariate Z^c, where ^{c} stands for *covariate*), that is:

$$\pi(x|z^c). \tag{1}$$

These marginal probabilities of being in a specific class given a value on the covariate, add to 1 over the latent classes x:

TABLE 4
Input data file for the latent class regression analysis; 16 lines per case, each line representing a transitive reasoning task

Replication	*Case Id*	*Grade*	*Format**	*Presentation**	*Content**	*Strategy*
1	1	2	1	1	1	3
2	1	2	2	1	1	2
3	1	2	3	1	1	6
4	1	2	4	1	1	2
5	1	2	1	2	1	7
6	1	2	2	2	1	6
.	1	2	.	.	.	.
.	1	2	.	.	.	.
.	1	2	.	.	.	.
15	1	2	3	2	2	5
16	1	2	4	2	2	3
1	2	3	1	1	1	5
2	2	3	2	1	1	1
.	.	3	.	.	.	.
16	2	3	4	2	2	6
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
1	615	6	1	1	1	4
.	615	6	.	.	.	.
.	615	6	.	.	.	.
16	615	6	4	2	2	3

*Format, presentation, and content were the three task characteristics; for a detailed description see Bouwmeester & Sijtsma (2003b).

$$\sum_{x} \pi(x|z^c) = 1. \qquad (2)$$

In the second part of the model, the probabilities are estimated of using a particular cognitive behaviour given the latent class and the value(s) on one or more external conditions. In this application the dependent variable “cognitive behaviour” is the discrete variable “strategy” (Y, with realizations y) that has seven categories. The predictor variables “external conditions” are three “task characteristics” ($Z^p{}_1$, $Z^p{}_2$, $Z^p{}_3$, with realizations $z^p{}_1$, $z^p{}_2$, $z^p{}_3$ (where ^{p} stands for *predictor*) having also a discrete measurement level:

$$f\left(y|x, z^p_1, z^p_2, z^p_3\right). \qquad (3)$$

For each task (which consists of a combination of the three task characteristics) a multinomial probability function is estimated for the use of a strategy in a latent class, and this is done for each combination of a strategy and a latent class. In a fixed latent class, these probabilities add to 1 over strategies (y), that is:

$$\sum_{y} f(y|x, z_1^p, z_2^p, z_3^p) = 1. \tag{4}$$

Because there are 16 tasks, there are 16 of these probability functions for each latent class.

Then, Equations 1 and 3 combine into the latent class regression model. The model is defined by the summation over latent classes of products of the marginal probability of being in a latent class given the grade level, and the product of multinomial probabilities for each task (denoted t, 16 combinations of task characteristics):

$$f(y|z^c, z_1^p, z_2^p, z_3^p) = \sum_{x} \pi(x|z^c) \prod_{t} f(y_t|x, z_{1t}^p, z_{2t}^p, z_{3t}^p). \tag{5}$$

Because there are 16 observations per case, the dependent variable $\mathbf{Y}$ is a vector containing the sixteen strategy responses and the predictor variables $\mathbf{Z}_l^p$ are also vectors containing the levels of the task characteristics.

Parameters

To calculate the multinomial probabilities of being in a latent class given grade level ($\pi(x|z^c)$ in Equation 5), two kinds of parameters have to be estimated, denoted by γ_x^0 and $\gamma_{z^c x}^1$. Parameters γ_x^0 are the intercepts for the latent class variable and parameters $\gamma_{z^c x}^1$ are the covariate effects on the latent class variable. Equation 1 is modelled by a multinomial probability, which is defined as a logistic regression function:

$$\pi(x|z^c) = \frac{\exp(\eta_{x|z^c})}{\sum_{x} \exp(\eta_{x|z^c})}. \tag{6}$$

The linear term $\eta_{x|z^c}$ equals

$$\eta_{x|z^c} = \gamma_x^0 + \gamma_{z^c x}^1 \tag{7}$$

To estimate the multinomial probability function of using a particular strategy given the latent class and a combination of task characteristics (i.e., $f(y_t|x, z_{1t}^p, z_{2t}^p, z_{3t}^p)$, in Equation 5), again two kinds of parameters have to be estimated, denoted by β_{xy}^1 and $\beta_{xz_{lt}^p}^2$. Parameters β_{xy}^1 are the class-specific intercepts. For all strategies in every latent class there is a β_{xy}^1 parameter.

Parameters $\beta^2_{xz^p_{lt}}$ are the class-specific regression coefficients. For all levels of the task characteristics there is a parameter for all strategies in every latent class. The multinomial probability function is again a logistic regression function:

$$f\left(y_t|x, z^p_{1t}, z^p_{2t}, z^p_{3t}\right) = \frac{\exp\left(\eta_{y|x,z^p_{1t},z^p_{2t},z^p_{3t}}\right)}{\sum_y \exp\left(\eta_{y|x,z^p_{1t},z^p_{2t},z^p_{3t}}\right)}. \tag{8}$$

The linear term $\eta_{y|x,z^p_{1t},z^p_{2t},z^p_{3t}}$ equals

$$\eta_{y|x,z^p_{1t},z^p_{2t},z^p_{3t}} = \beta^1_{xy} + \beta^2_{z^p_{1t}xy} + \beta^2_{z^p_{2t}xy} + \beta^2_{z^p_{3t}xy}. \tag{9}$$

The number of parameters to be estimated increases rapidly with an increasing number of latent classes. Table 5 shows the number of parameters to be estimated for models with one through seven latent classes.

Fit of the model

The program Latent Gold (Vermunt & Magidson, 2000, 2003) was used to estimate the parameters and calculate the fit of the model. The program gives evaluation statistics, estimates of the parameters and the accompanying standard errors and z-values.

In the program Latent Gold a number of evaluation statistics are provided to choose a plausible model. First, the log-likelihood statistics are calculated which express the fit for models with a user-specified number of latent classes. The amount of reduction of the log-likelihood statistic for models with an increasing number of classes can be considered to choose the best-fitting model. Because of sparse frequency tables, the asymptotic p-values associated with the χ^2-statistics often cannot be trusted. Therefore, a p-value can be estimated by means of bootstrapping (Efron & Tibshrani, 1993), which is implemented in the program. The bootstrap L^2 procedure involves generating a certain number of replication samples from the maximum likelihood solution and re-estimating the model with each replication sample. L^2 is a test statistic or fit measure. The bootstrap p-value is the proportion of replication samples with higher L^2 than in the original sample. For example, when 40% of the replication samples has an L^2 value higher than the L^2 value of the original sample, the bootstrap p-value is 0.40. However, a conditional bootstrap procedure, in which the fit of models with different classes can be compared has not yet been implemented in the Latent Gold program.

Second, the BIC values are calculated. The lower the BIC value, the better fitting and the more parsimonious the model (McLachlan & Peel,

TABLE 5
Number of parameters to be estimated

Classes	γ_x^0	$\gamma_{z^c x}^1$	β_{xy}^1	$\beta_{xz_{lt}^p}^2$	*Total*
1	$1 - 1 = 0$	$1 - 1 = 0$	$(7 - 1) \times 1 = 6$	$(7 - 1) \times ((4 - 1) + (2 - 1) + (2 - 1)) \times 1 = 30$	36
2	$2 - 1 = 1$	$2 - 1 = 1$	$(7 - 1) \times 2 = 12$	$(7 - 1) \times ((4 - 1) + (2 - 1) + (2 - 1)) \times 2 = 60$	74
3	$3 - 1 = 2$	$3 - 1 = 2$	$(7 - 1) \times 3 = 18$	$(7 - 1) \times ((4 - 1) + (2 - 1) + (2 - 1)) \times 3 = 90$	112
4	$4 - 1 = 3$	$4 - 1 = 3$	$(7 - 1) \times 4 = 24$	$(7 - 1) \times ((4 - 1) + (2 - 1) + (2 - 1)) \times 4 = 120$	150
5	$5 - 1 = 4$	$5 - 1 = 4$	$(7 - 1) \times 5 = 30$	$(7 - 1) \times ((4 - 1) + (2 - 1) + (2 - 1)) \times 5 = 150$	188
6	$6 - 1 = 5$	$6 - 1 = 5$	$(7 - 1) \times 6 = 36$	$(7 - 1) \times ((4 - 1) + (2 - 1) + (2 - 1)) \times 6 = 180$	226
7	$7 - 1 = 6$	$7 - 1 = 6$	$(7 - 1) \times 7 = 42$	$(7 - 1) \times ((4 - 1) + (2 - 1) + (2 - 1)) \times 7 = 210$	264

2000). Third, the proportions of classification errors are provided. This proportion indicates how well the model can predict latent class membership given the value on the covariate and the dependent variable (Andrews & Currim, 2003). This proportion is not a fit measure, but it is an important measure to evaluate the distinctiveness of different classes.

Fourth, the class sizes and the interpretation of the classes were used to choose a model. Although the evaluation statistics calculated by the program provided useful guidelines to choose the best-fitting model, the final decision was based on the interpretability of the classes and the class size.

RESULTS

Analysis of variance, with number-correct score on all 16 tasks as dependent variable and school and grade as independent variables, showed no significant effects for the same grades of different schools. Therefore, it was concluded tentatively that school had no influence on a child's transitive reasoning ability.

Model fit and number of classes

Seven models were fitted with an increasing number of classes ranging from one to seven. Table 6 shows the evaluation statistics that were used to choose a final model.

Although a number of fit-statistics that evaluate different aspects of the model can be used to choose a plausible model, the choice of a final model also depends on substantive considerations, previous research results, considerations of parsimony, and so on. This can be compared with factor analysis, where the choice of the final factor solution also

TABLE 6
Model fit statistics for seven latent class models

Number of classes	*L^2 value*	*BIC value*[a]	*Number of parameters*	*Proportion of classification errors*
1	25338.593	31320.685	36	0.000
2	22867.005	29093.119	74	0.028
3	21824.833	28294.969	112	0.046
4	21176.725	27890.430	150	0.053
5	20777.859	27736.038	188	0.050
6	20430.206	27632.407	226	0.057
7	20204.839	27651.062	264	0.060

[a]BIC value = -2 log-likelihood + No. parameters $*$ ln(N).

depends on considerations other than statistical ones. It remains difficult, maybe practically impossible, to determine the exact number of latent classes.

The log-likelihood statistics showed a reduction of at least 37% in magnitude from the log-likelihood statistics going from the One-Class model to the Five-Class model. The reduction of the log-likelihood statistic from the Five-Class model to the Six-Class model was only 12%. On the basis of the log-likelihood statistics the Five-Class model would be chosen.

The Six-Class model was the most parsimonious model in terms of BIC values. The proportion of classification errors first increased from the One-Class model through the Four-Class model. This can be explained by the fact that correct classifying is more difficult with a higher number of classes. The result that the proportion of classification errors decreases with the Five-Class model and then increases again indicates that the Five-Class model may be preferred over the Six-Class model.

On the basis of the evaluation statistics provided by the program, the Five- and Six-Class models are most plausible. For this application, the bootstrap procedure was not informative about choosing the best-fitting model. On the basis of the class sizes and the interpretation of the classes, the Five-Class model was chosen as the final model. The Six-Class model had three relative small classes (marginal probability < 0.10). Moreover, the smallest class hardly differed from another class with respect to the interpretation.

The γ-parameters: class size and influence of grade

Table 7 shows the γ parameters. The γ^0-parameters are intercept parameters, which are used to calculate class size. The $\gamma^1_{z^c x}$-parameters were all significant ($z > 1.96$). This means that the covariate grade had a significant influence in all classes. When these γ-parameters are inserted in Equations 7 and 6, respectively, the marginal probabilities (class size, see Table 7) and the probability distribution of grade given the latent class can be calculated.

TABLE 7
γ-Parameter estimates and class size for the five-class model

Class	γ^0_x	$\gamma^1_{z^c x}$	*Class size*
1	− 0.251	0.220	0.381
2	− 4.386	0.796	0.266
3	− 1.866	0.325	0.146
4	3.957	− 0.812	0.106
5	2.545	− 0.530	0.101

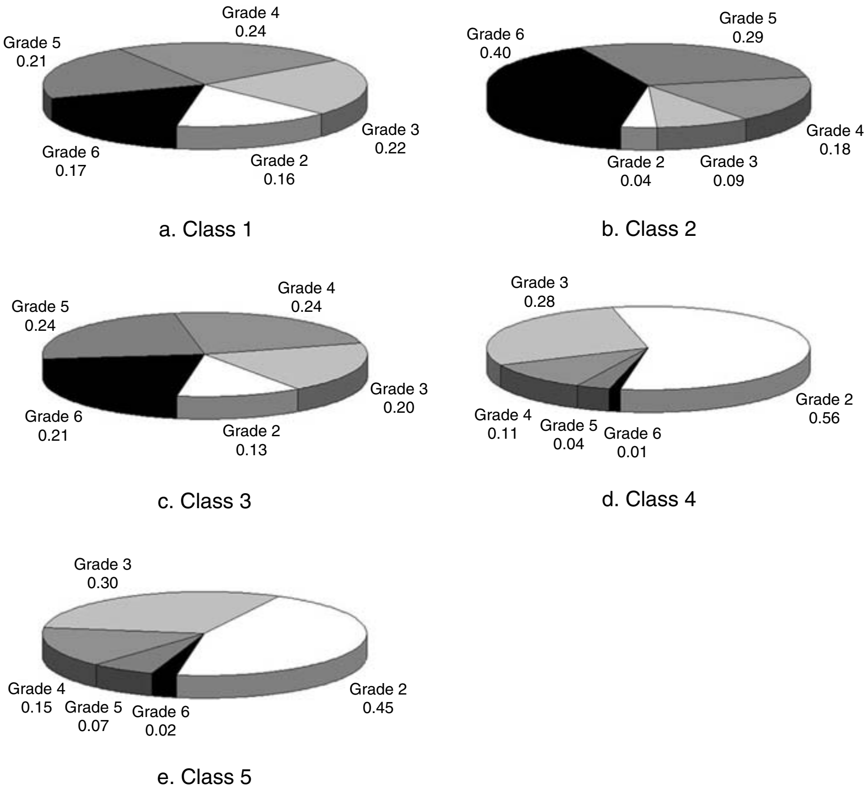

Figure 2. Probability distribution of grade given latent class.

Figure 2 shows the probabilities of grade for each class. In particular, in Class Two, Grade Six had high probability. Also in Classes One and Three, higher grades had higher probability than lower grades. For the Classes Four and Five, lower Grades Two and Three had higher probability than higher Grades Four, Five and Six.

The β parameters: strategy use and influence of task characteristics

Table 8 shows the class-specific intercepts, the β^1_{xy}-parameters. A non-significant β^1_{xy}-parameter estimate does not significantly deviate from zero, which means that there is no effect for this strategy in the particular class. The β^1_{xy}-parameters can be used to calculate the probability distribution of strategy given the latent class,

$$\pi(y|x) = \frac{\exp(\beta^1_{xy})}{\sum_y \exp(\beta^1_{xy})}.$$

Figure 3 shows the probabilities of using a particular strategy for each class.

Children in Class One in particular use INCORRECT premise information, LITERAL premise information and NO EXPLANATION. Children in Class Two are characterized by the use of LITERAL premise information and INCORRECT premise information. Children in Class Three are characterized by the use of INCOMPLETE premise information and INCORRECT premise information. Children in Class Four in particular do NOT give an EXPLANATION or use EXTERNAL & VISUAL information. Children in Class Five are characterized by the use of EXTERNAL & VISUAL information, FALSE MEMORY and NO EXPLANATION.

There are 280 class-specific regression coefficient parameters ($\beta^2_{xz^p_{lt}}$), that is, one for each strategy (7) in each class (5), for every level of the task characteristics (4 + 2 + 2). It is beyond the scope of this article to interpret these parameters in detail, but we will give a global interpretation of the influence of the task characteristics on the strategy use in the latent classes by describing the size of the effect of the parameters. Table 9 gives the interpretation of the strength of the influences. TASK FORMAT has some influence on strategy use in the Classes One and Two but hardly in the Classes Three, Four and Five. PRESENTATION FORM has a strong effect on strategy use in four Classes but not in Class Three. CONTENT OF THE RELATION has a strong effect on strategy use in the Classes One and Two, some effect in the Classes Three and Four and hardly any effect in Class Five.

TABLE 8
The β_{xy}-parameter estimates for the five-class model

STRATEGY	*Class 1*	*Class 2*	*Class 3*	*Class 4*	*Class 5*
LITERAL	0.705	2.260	*0.788*	– 1.137	– 0.910
REDUCED	*1.421*	*– 0.026*	*– 0.871*	*– 2.320*	*– 2.351*
INCORRECT	1.098	1.900	1.707	*0.433*	*0.038*
INCOMPLETE	– 0.704	*0.392*	2.559	*– 0.770*	*– 0.651*
FALSE MEMORY	*– 0.273*	– 2.367	*0.409*	*0.440*	0.887
EXTERNAL & VISUAL	*– 0.236*	– 0.926	*0.410*	*0.726*	2.040
NO EXPLANATION	0.831	– 1.232	*– 5.001*	2.629	0.946

Italics: effect is not significant ($p > .05$).

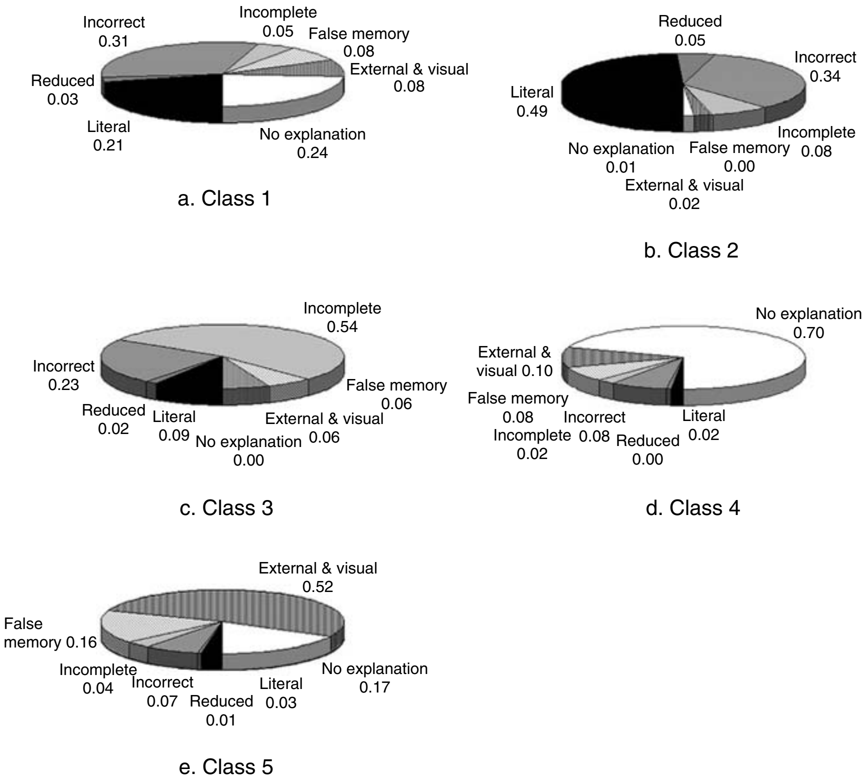

Figure 3. Probability distribution of strategy by latent class.

TABLE 9
Size of the effect of influence of the task characteristics on strategy use

CHARACTERISTIC	*Class 1*	*Class 2*	*Class 3*	*Class 4*	*Class 5*
TASK FORMAT	some	some	hardly	hardly	hardly
PRESENTATION FORM	strong	strong	hardly	strong	strong
CONTENT	strong	strong	some	some	hardly

DISCUSSION

When studying cognitive development of transitive reasoning using a latent class regression model we found that a number of classes can be distinguished, which differ with respect to cognitive behaviour. Using the grade level distribution over the classes, an ordering of the classes became visible. Classes Four and Five, which contained in particular lower-grade children, were characterized by superficial cognitive behaviour using almost no task information but rather directly observable task characteristics or unimportant information from the external world. Several times, children gave no explanation at all. In the Classes One and Three, in which children from all grades were represented but in particular from Grades Three, Four, and Five, children often knew that they had to use the task information but they did not have a complete or correct representation of the task space. Class Two contained in particular higher-grade children who were able to use the task information, understand the underlying pattern and form a complete internal representation of the task space in most cases.

By treating age as a covariate, which influenced the formation of the classes, the developmental ordering of the classes was not assumed to be known a priori. The results showed that there is a developmental ordering, but that children from the highest grades are (marginally) represented in lower-ability classes, while children from lower grades are represented in the higher-ability class. The model thus gives opportunities to diagnose children who deviate from the age-related criterion and to interpret that deviation in detail.

An interesting finding of this application, which is difficult to reveal when no latent classes are distinguished, is the differential influence of task characteristics on strategy use in a latent class. In addition to a general overview of the strategy use in a particular latent class, the latent class regression model makes it possible to explain or predict the influence of external conditions at a detailed level, or even the influence of interactions of external conditions (which was not done in this application).

An interesting product of this method is that specific cognitive behaviour can be better interpreted in relation to other cognitive behaviour. For example, it is difficult to interpret the NO EXPLANATION strategy when there is no further information. When children do not give an explanation, they may simply not know how to solve the problem; they may know that the premises information has to be used, but they do not know how; or they may simply not know how to explain their answer to other people. The distribution of the strategies over the classes gives information on how to interpret this NO EXPLANATION strategy. In Class One and Class Four children often used NO EXPLANATION, but children in Class One used some more-proficient strategies besides the NO EXPLANATION strategy, while children in Class Four in particular used low-proficiency strategies. It appears that children in Class Four had absolutely no idea how to solve the tasks, while children in Class One understood that they had to use the premises but did not know how to use them.

The analysis of this application was explorative. We did not assume a particular cognitive theory which was tested. However, it is also possible to test a cognitive theory in terms of the latent class regression model, that is, to perform a confirmative analysis. Assuming Piaget's theory, we could have tested whether empirical data fitted in the cognitive developmental stages Piaget assumed. Then, it should be tested whether the data could be explained by a number of latent classes, which represented the cognitive developmental stages. Using the latent class regression model, it is also possible to model a priori hypotheses about developmental stages. It may be expected from a developmental theory that a particular condition has no effect in one particular latent class, or an equal effect in two or more different classes. By imposing restrictions on the model, effects can be set to zero, or can be set equal for different classes.

It has to be emphasized that we used data from a cross-sectional design, that is, children of different ages were tested once. This design makes it possible to interpret development in terms of differential classes, but we can only speculate about an individual child's transition from one class to another. A longitudinal study is necessary to study this transition. The latent class regression model can also be used to study such a longitudinal design.

In this article we introduced the latent class regression model for studying cognitive developmental phenomena. The most important value of the model is the possibility it provides to empirically test the presence or absence of latent classes without the need for strong cognitive theoretical assumptions about the latent variable(s). In the application of the model to transitive reasoning data, a very large number of parameters had to be estimated, making the model relatively complex. The large number of parameters was caused by a nominal dependent variable, having seven categories, and nominal independent variables. Models with other types of dependent and independent variables will contain substantially fewer parameters.

The flexibility of the model in terms of mixed measurement levels and treatment of different cognitive variables further offers a broad application to a number of cognitive developmental phenomena, such as conservation, symbolic analogies, verbal analogies, inductive reasoning, reading comprehension, and problem solving.

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