Logit Equations for Latent Class Model Probabilities

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Introduction

- LC model probabilities are typically modelled using a (multinomial) logistic regression parameterization.
- In Latent GOLD these appear in the Parameters output.

- In this video I will explain:
  - why we use this parameterization
  - how the logit parameters are related to the probabilities appearing in Profile
  - how to interpret these parameters depending on the coding used

Why using this logistic parameterization?

- ML estimation is much easier without range constraints: logit parameters can take on values between minus and plus infinity, while probabilities are restricted to be between 0 and 1.
- Inference (z-tests and Wald tests) for unbounded parameters are more reliable.
- It allows for interesting extensions, such as models with local dependencies, models for ordinal indicators, models with multiple latent variables, and mixture regression and mixture growth models.

Logit equation for response probabilities

$$P(y_j \mid X = c) = \frac{\exp(\alpha^j_{y_j} + \beta^j_{y_j c})}{\sum_{m=1}^{M_j} \exp(\alpha^j_m + \beta^j_{mc})}$$

- $\alpha^j_{y_j}$ is the intercept parameter for a category of item *j*
- $\beta^j_{y_j c}$ is the slope parameter of class *c* for a category of item *j*

Logit equation for class proportions

$$P(X = c) = \frac{\exp(\gamma_{0c})}{\sum_{c'=1}^{C} \exp(\gamma_{0c'})}$$

- In a simple LC model this equation contains only the intercepts γ_{0c}
- However, more extended models may include covariates affecting the classes, or multiple latent variables affecting one another

Parameters under effect & dummy coding

Models for Indicators						
	Cluster1	Cluster2	Cluster3	Wald	p-value	R^2
accuracy						
mostly true	0.7450	0.8501	-1.5951	5.8756	0.053	0.2047
not true	-0.7450	-0.8501	1.5951			
cooperat						
interested	1.9487	-0.9131	-1.0356	12.0583	0.017	0.1346
cooperative	0.4051	-0.0532	-0.3520			
Impatient,Hostile	-2.3538	0.9663	1.3875			
understa						
Good	1.4306	-1.0983	-0.3323	10.2440	0.0060	0.4013
Fair/Poor	-1.4306	1.0983	0.3323			
purpose						
GOOD PURPOSE	0.7676	0.9045	-1.6721	35.6370	3.4e-7	0.3373
DEPENDS	-0.3144	-0.0620	0.3764			
WASTE OF TIME AND $	-0.4532	-0.8425	1.2957			
Intercepts	Overall	Wald	p-value			
accuracy						
mostly true	-0.5508	2.2925	0.13			
not true	0.5508					
cooperat						
interested	1.9533	53.9749	1.9e-12			
cooperative	0.3263					
Impatient,Hostile	-2.2796					
understa						
Good	0.8509	1.9512	0.16			
Fair/Poor	-0.8509					
purpose						
GOOD PURPOSE	1.0637	39.2610	3.0e-9			
DEPENDS	-0.6106					
WASTE OF TIME AND $	-0.4531					
Model for Clusters						
Intercept	Cluster1	Cluster2	Cluster3	Wald	p-value	
	0.6585	-0.1180	-0.5405	35.6042	1.9e-8	

Models for Indicators						
	Cluster1	Cluster2	Cluster3	Wald	p-value	R^2
accuracy						
mostly true	0.0000	-0.0000	-0.0000	5.8756	0.053	0.2047
not true	-0.0000	-0.2102	4.6801			
cooperat						
interested	0.0000	-0.0000	-0.0000	12.0583	0.017	0.1346
cooperative	-0.0000	2.4034	2.2272			
Impatient,Hostile	-0.0000	6.1818	6.7256			
understa						
Good	0.0000	-0.0000	-0.0000	10.2440	0.0060	0.4013
Fair/Poor	-0.0000	5.0578	3.5257			
purpose						
GOOD PURPOSE	0.0000	-0.0000	-0.0000	35.6370	3.4e-7	0.3373
DEPENDS	-0.0000	0.1156	3.1306			
WASTE OF TIME AND $	-0.0000	-0.5262	4.1886			
Intercepts	Overall	Wald	p-value			
accuracy						
mostly true	-0.0000	16.9581	3.8e-5			
not true	-0.3884					
cooperat						
interested	-0.0000	23.9399	6.3e-6			
cooperative	-3.1704					
Impatient,Hostile	-8.5353					
understa						
Good	-0.0000	1.5866	0.21			
Fair/Poor	-4.5630					
purpose						
GOOD PURPOSE	-0.0000	271.2944	1.2e-59			
DEPENDS	-2.7564					
WASTE OF TIME AND $	-2.7375					
Model for Clusters						
Intercept	Cluster1	Cluster2	Cluster3	Wald	p-value	
	-0.0000	-0.7764	-1.1990	35.6042	1.9e-8	

Computation of Profile using Parameters

- Effect coding

$$P(Accuracy = 1 | X = 2) = \frac{\exp(\text{-}0.5508 + 0.8501)}{\exp(\text{-}0.5508 + 0.8501) + \exp(0.5508\text{-}0.8501)} = 0.6453$$

- Dummy first coding

$$P(Accuracy = 1 | X = 2) = \frac{\exp(0 + 0)}{\exp(0 + 0) + \exp(\text{-}0.3884\text{-}0.2102)} = 0.6453$$

Final remarks

- In simple applications on LC analysis, you don’t need to worry about the logit parameters. Looking at Profile suffices.
- However, for extensions of the basic model, we need the logistic parameterization.
- Profile output is directly linked to the logit parameters.
- Under effect coding, the parameters indicate whether particular indicator-cluster combinations are more (or less) likely than average.
- Under dummy coding, the parameters are log-odds ratios. These can be exponentiated to obtain odds-ratios.