

# Latent Trait Models: An Introduction

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# Introduction

- In this video I will discuss models that can be used when the indicators are categorical (typically dichotomous or ordinal) and when the underlying latent variable can be assumed to be continuous.
- While Latent GOLD is not specialized in latent trait or IRT modeling, I will show how it can be used to estimate such models.

# Four-fold typology of latent variable models

- Latent trait model as one of the types of latent variable models

Manifest Variables / Observed indicators	Latent Variable(s)	
	Continuous	Categorical
Continuous	Factor analysis	Latent profile analysis
Categorical	Latent trait model / Item response theory (IRT)	Latent class analysis

# Example: Hospital Anxiety and Depression Scale (HADS)

- See: [https://en.wikipedia.org/wiki/Hospital\\_Anxiety\\_and\\_Depression\\_Scale](https://en.wikipedia.org/wiki/Hospital_Anxiety_and_Depression_Scale)
- Anxiety items:
  1. I feel tense or wound up
  2. I get a sort of frightened feeling as if something awful is about to happen
  3. Worrying thoughts go through my mind
  4. I can sit at ease and feel relaxed
  5. I get a sort of frightened feeling like 'butterflies' in the stomach
  6. I feel restless as I have to be on the move
  7. I get sudden feelings of panic
- These have 4 categories, scored 0-3 for negatively and 3-0 for positively formulated items. I will use a version with dichotomous items obtained by merging categories 0-1 and 2-3.

# Example: HADS

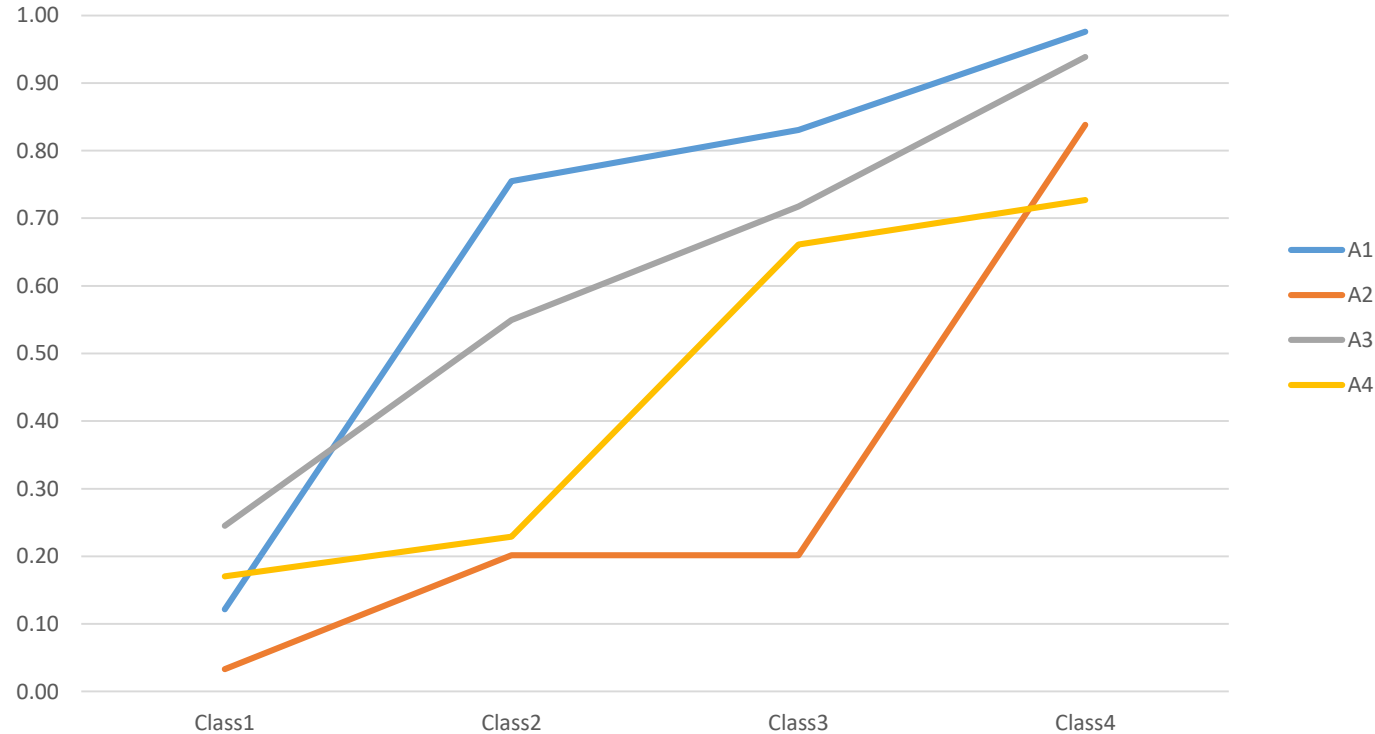
- Results of a LC analysis with this data set:

		LL	BIC(LL)	AIC(LL)	AIC3(LL)	Npar	L <sup>2</sup>	df	p-value	Max. BVR
<b>Model1</b>	1-Cluster	-3930.1412	7907.6054	7874.2825	7881.2825	7	1014.9129	120	1.4e-141	151.1198
<b>Model2</b>	2-Cluster	-3529.3048	7160.0159	7088.6097	7103.6097	15	213.2402	112	2.7e-8	15.4561
<b>Model3</b>	3-Cluster	-3493.2005	7141.8905	7032.4010	7055.4010	23	141.0314	104	0.0092	12.0338
<b>Model4</b>	4-Cluster	-3481.6546	7172.8820	7025.3092	7056.3092	31	117.9396	96	0.064	2.1043
<b>Ordered</b>	4-Cluster	-3483.8683	7157.0282	7023.7366	7051.7366	28	122.3671	99	0.056	2.0214

- In the 1, 2, and 3-class models, classes come out as ordered. In 4-class model not, but we can force this to happen using the “Order Restricted Clusters” option on the Model tab.

# Example: IRFs for order-restricted LC model

- Profile-like plot for A1, A2, A3, and A4, but with classes on x-axis and lines corresponding to items instead of classes. How can you see that classes ordered?



# Example: HADS

- The order-restricted LC model seems to perform well, indicating the item measure a single dimension.
- But what about using a continuous specification for the latent variable?
- Does it makes sense to assume that every person has its own score on “anxiety”, rather than that there are “only” 4 possible levels?
- The latter is what is done in latent trait or IRT models.

# Differences & similarities between latent class and latent trait models

- In a LC (or latent profile) analysis, we identify qualitatively different subgroups.
- Depending on the data, the latent classes may come out as ordered. When this happens, this indicates the indicators measure a single dimension. As we saw, we may also “force” the classes to be ordered.
- In IRT modeling, we *assume* the items/indicators measure a single dimension. This is referred to as the *assumption of uni-dimensionality*.
- Moreover, instead of looking for subgroups, the aim is to *construct a quantitative scale*. The latter implies that the latent variable is treated as continuous. In most texts it is denoted by the Greek symbol “theta” ( $\theta$ )



# IRT model equations

- As LC analysis, IRT assumes *local independence*, or independence conditional on the latent variable “score”  $\theta$  :

$$P(y_1, \dots, y_J | \theta) = \prod_{j=1}^J P(y_j | \theta)$$

- Logit equations define the response probabilities (or *item response functions*) of the  $J$  items/indicators. For dichotomous indicators and using dummy coding, we get:

$$P(y_j = 1 | \theta) = \frac{\exp(\alpha^j + \beta^j \cdot \theta)}{1 + \exp(\alpha^j + \beta^j \cdot \theta)}$$

# IRT model equations

- This model is called the 2-parameter logistic (2-PL) model, and is usually defined in a slightly different manner:

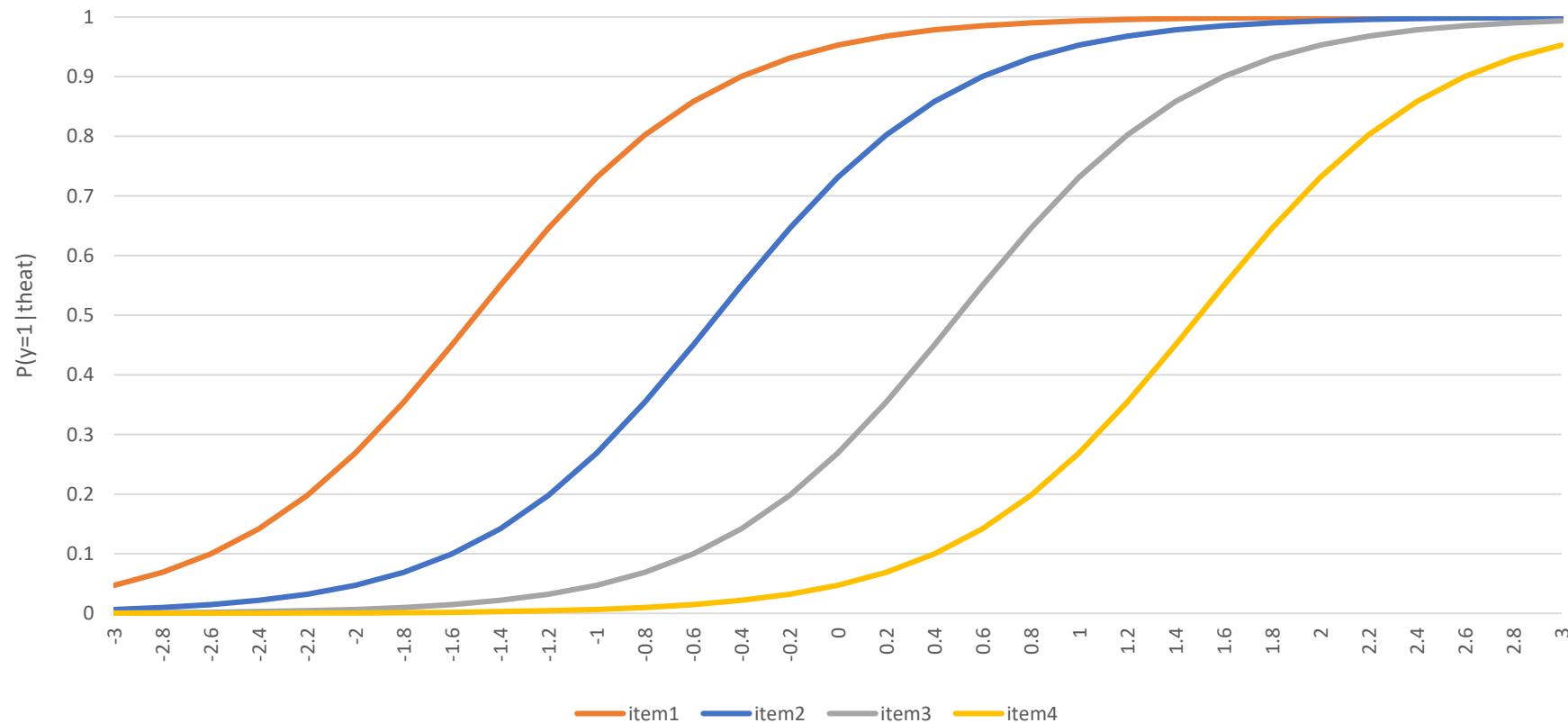
$$P(y_j = 1 | \theta) = \frac{\exp[a^j \cdot (\theta - d^j)]}{1 + \exp[a^j \cdot (\theta - d^j)]}$$

- Here,  $d^j$  and  $a^j$  are called difficulty and discrimination parameter. These are related as follows to the intercept and slope parameters:

$$\begin{array}{lcl} \beta^j = a^j & \text{or} & a^j = \beta^j \\ \alpha^j = -a^j \cdot d^j & & d^j = -\alpha^j / \beta^j \end{array}$$

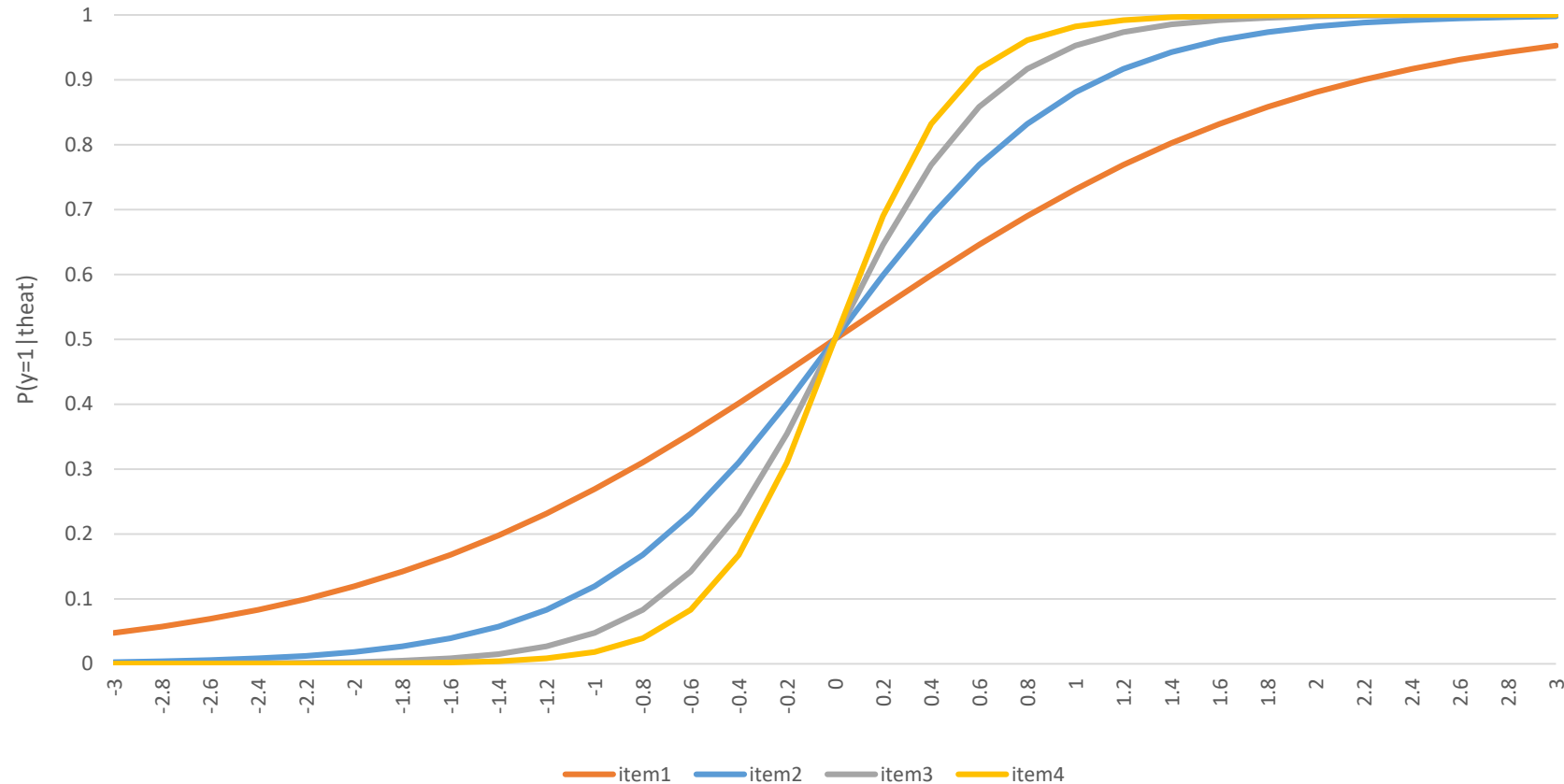
# Item response functions (IRFs)

- IRFs with difficulty parameters equal to -1.5, -.5, .5 and 1.5:



# Item response functions (IRFs)

- IRFs with discrimination parameters equal to 1, 2, 3, and 4:



# Item response functions (IRFs)

- What can you see if you change the difficulty parameter? What is the interpretation of the difficulty parameter?
- What can you see if the discrimination parameter becomes larger/smaller?
- Note that the location and the scale of the latent variable is undefined. For parameter estimation using maximum likelihood and interpretation, we typically assume the latent variable to be normally distributed, with mean=0 and variance=1.

# IRT modeling with Latent GOLD

- Latent GOLD does not have a special IRT module, but it can estimate the most important types of IRT models
- Simplest approach:
  - 1-Cluster model with 1 CFactor (see Advanced tab)
  - Dummy first coding (Output tab)
- Alternative:
  - Use the Latent GOLD Syntax to specify a model with a continuous latent variable
- Parameters output gives the *alpha* and *beta* parameters. From these you can compute the discrimination and difficulty parameters.
- BIC, AIC, AIC3, L2, X2, and BVRs are relevant in IRT as well. Specialized IRT programs also provide fit measures for items, and some even for persons.

# Example: HADS

- Results obtained with HADS data set:

		LL	BIC(LL)	AIC(LL)	AIC3(LL)	Npar	L <sup>2</sup>	df	p-value	Max. BVR
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<b>2-PL model</b>	1-Cluster 1-CFactor	-3496.0463	7086.7384	7020.0926	7034.0926	14	146.7231	113	0.018	11.0361

- The IRT model is more parsimonious, and therefore preferred over the LC models by the information criteria
- It seem to have a minor issue with a local dependency (between A4 and A6)

# Example: HADS

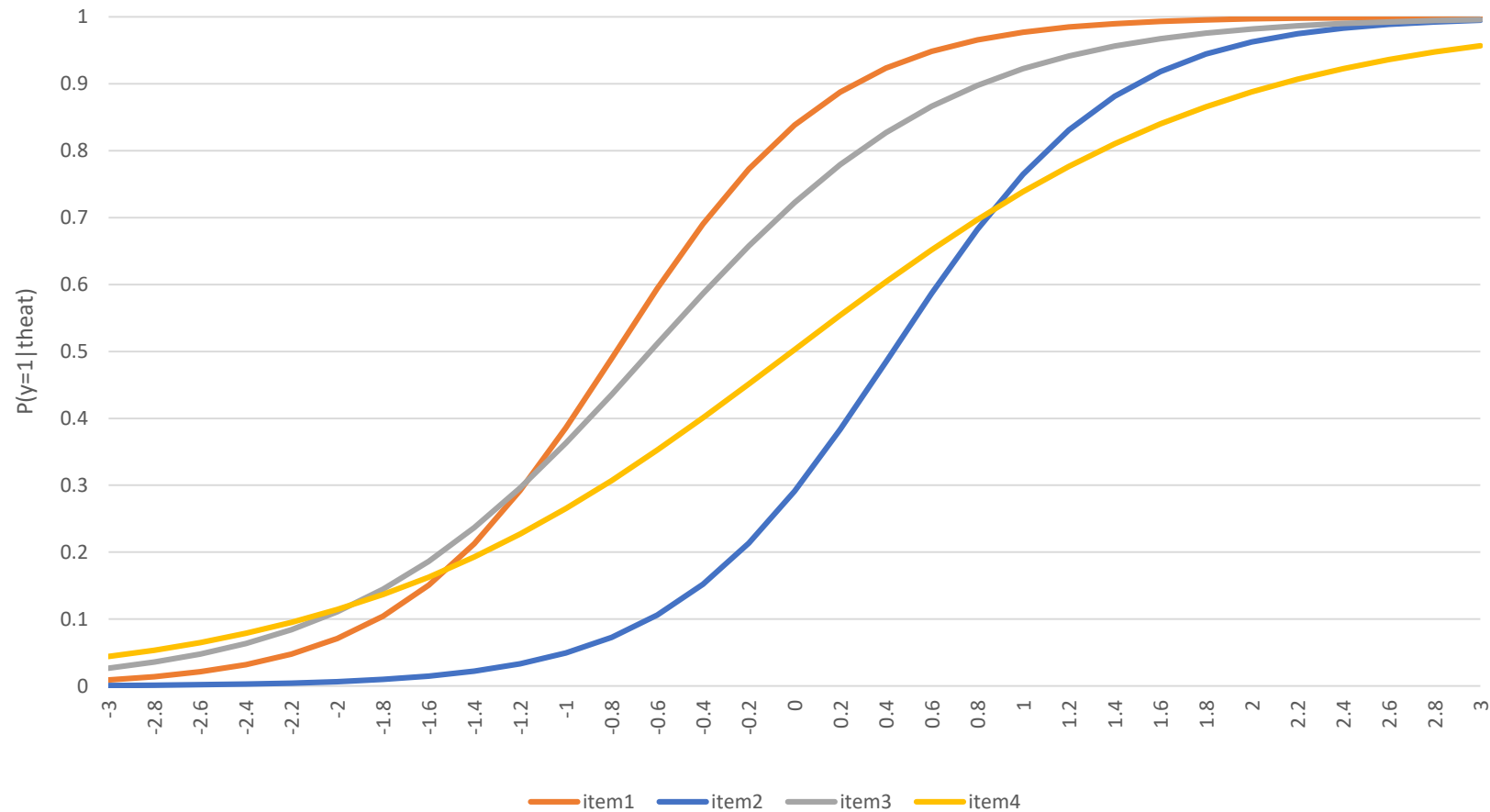
- beta (or a), alpha and d parameters obtained with the HADS data set:

	beta / a	alpha	d
A1	2.11	1.65	-0.78
A2	2.07	-0.89	0.43
A3	1.52	0.97	-0.63
A4	1.03	0.01	-0.01
A5	1.75	-0.74	0.43
A6	1.11	0.64	-0.58
A7	1.72	-0.88	0.51



# Example: HADS

- Graphical display of IRFs of A1-A4



# Final remarks

- Other IRT models than the 2-PL model exist:
  - Dichotomous items: 1-PL and 3-PL model
  - Ordinal items: partial credit and graded response models
  - Nominal items: nominal response model
- As we saw in LC modeling:
  - We may get an estimate of the latent variable scores and their uncertainties
  - We may investigate the relationship with covariates and distal outcomes
- Other issues:
  - Differential item functioning: when a covariate affects the items directly
  - Multidimensionality: multiple latent variables (thetas)